

# Lecture Notes

## International Financial Crises

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### Abstract

These lecture notes cover currency crises, banking crises and debt crises, and include quite a bit on expectations and coordination. Its main focus is on theoretical models, but the last two sections are on empirical methods that can be used to shed light on these (and other) issues. If you happen to find them useful for your teaching or to study any of these topics, feel free to use it – and please send me an email to let me know. Comments, suggestions, and corrections of any mistakes or typos are more than welcome.

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# 1 Currency crises: basic models

## 1.1 The first generation models of currency crises

Salant and Henderson (1978) showed that if the government uses a stockpile of an exhaustible resource (e.g., gold) to stabilize its price, eventually a speculative attack will occur: the private investors will suddenly acquire the entire government's stock.

Paul Krugman (1979) showed that if a pegged exchange rate coexists with budget deficits that need to be financed by money creation, the argument in Salant and Henderson (1978) also applies: a speculative attack will force the government to abandon the pegged regime.

Flood and Garber (1984) develops the concept of "shadow exchange rate" and provide two linear examples of the logic presented by Krugman (1979). This notes covers the deterministic model of that paper. The stochastic model of Flood and Garber (1984) is also interesting.

### 1.1.1 Flood and Garber (1984)

The exchange rate is denoted by  $S$ . In the non-stochastic version of Flood and Garber (1984), the exchange rate is initially pegged at  $\bar{S}$ . Money demand depends negatively on interest rates:

$$\frac{M_t}{P_t} = a_0 - a_1 i_t \quad (1)$$

Money supply equals foreign currency reserves ( $R_t$ ) plus domestic credit ( $D_t$ ).

$$M_t = R_t + D_t \quad (2)$$

Domestic credit is expanding:

$$\frac{\dot{D}_t}{D_t} = \mu, \mu > 0 \quad (3)$$

The model assumes there will be no arbitrage opportunities in markets for goods and assets. That implies interest rate parity (IRP) and purchasing power parity (PPP) ( $i_t^*$  and  $P_t^*$  are constants).

$$P_t = S_t \cdot P_t^* \quad (4)$$

$$i_t = i_t^* + \frac{\dot{S}}{S} \quad (5)$$

Initially, the government has a positive stock of reserves and will keep the peg until reserves reach a given minimum level (say, until  $R_t = 0$ ). Before the peg is abandoned,  $\dot{S} = 0$ . By PPP (equation 4),  $\dot{P} = 0$ , and by IRP (equation 5),  $i_t$  is constant, equal  $i_t^*$ .

Therefore  $M_t$  is also constant (equation 1). Define  $M^H$  as the demand for money while the peg is kept:

$$\frac{M^H}{\bar{S} \cdot P_t^*} = (a_0 - a_1 i_t^*) \quad (6)$$

In the model, the expansion of domestic credit generates loss of reserves until the moment in which the peg is abandoned. Then, it leads to an increasing trend in the money supply and, consequently, inflation. Therefore, after the peg is abandoned, the demand for real balances is smaller because the nominal interest rate is higher, due to inflation (equations 5 and 1). An arbitrage condition implies that  $P_t$  and  $S_t$  cannot jump up, and so the discrete reduction in money demand translates in a discrete fall of  $M_t$ . Initially, reserves are falling steadily, at a rate  $\mu$ . When  $R_t$  is exactly equal to the difference in money demand in both regimes, all agents exchange part of their domestic currency for foreign currency and the government is forced to abandon the peg. Define  $M^L$  as the demand for money right after the peg is attacked:

$$m^L = \frac{M^L}{\bar{S} \cdot P_t^*} = \left( a_0 - a_1 \left( i_t^* + \frac{\dot{S}}{\bar{S}} \right) \right) \quad (7)$$

Now, define the shadow exchange rate ( $\tilde{S}_t$ ) as the exchange rate that would prevail if the currency was allowed to float (demand for real balances would be  $m^L$ ) and foreign reserves vanished (so that  $M_t = D_t$ ). PPP implies that  $P_t = P_t^* \tilde{S}_t$  and we have:

$$m^L = \frac{M_t}{P_t} = \frac{D_t}{\tilde{S}_t P_t^*} \quad (8)$$

Equations 7 and 8 imply:

$$\tilde{S}_t = \frac{\bar{S}}{M^L} D_t \quad (9)$$

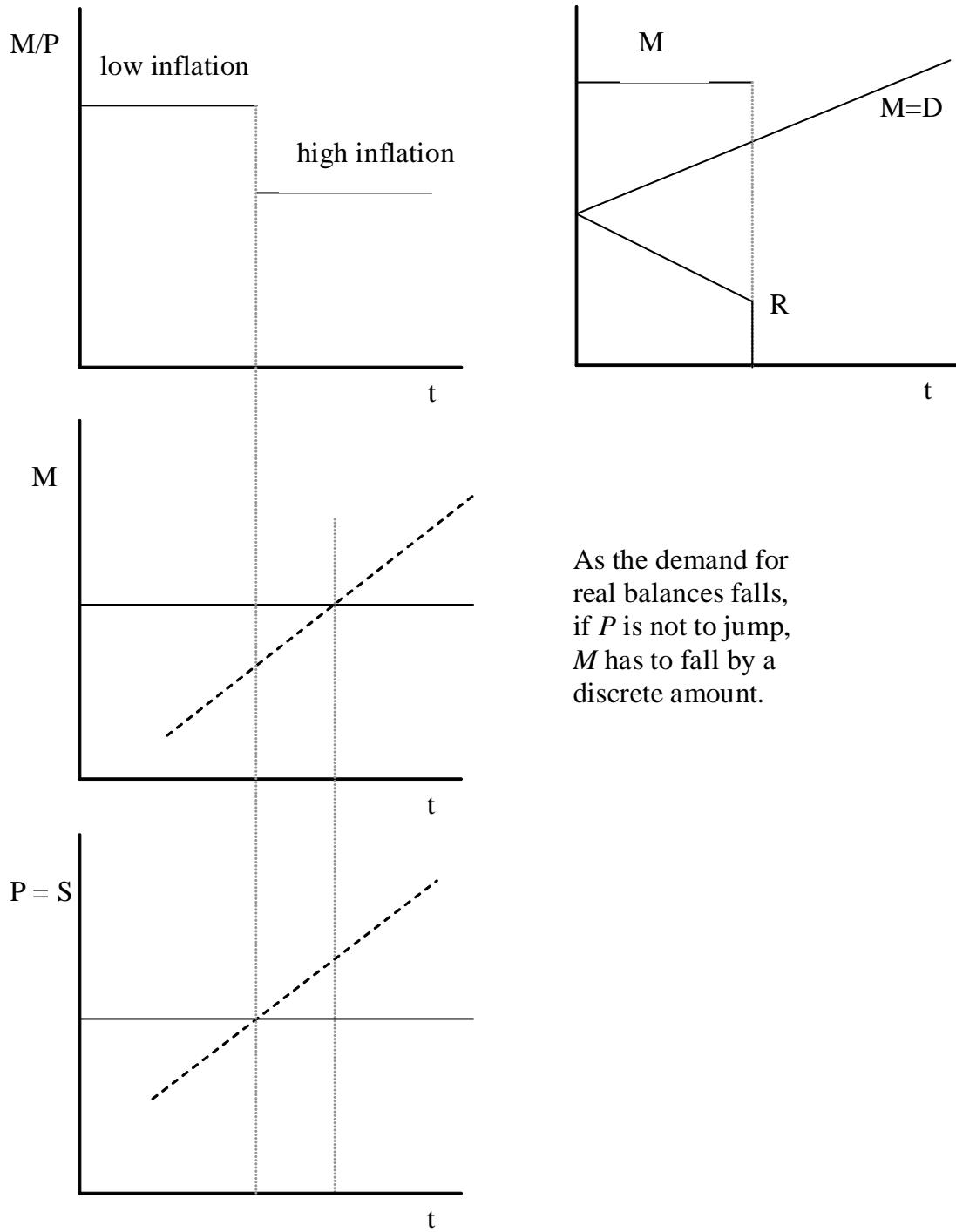
As Flood and Garber (1984) show, a speculative attack forces the abandonment of the peg exactly when  $\tilde{S}_t = \bar{S}$ .

**The size of the attack** In Flood and Garber (1984), a speculative attack is an instantaneous event: agents exchange some of their local currency for foreign currency ( $M$  falls) and deplete the Central Bank stock of reserves ( $R$  falls to 0). What is the lost in reserves?

Right before the attack, we have:

$$M^H = \bar{S} \cdot P_t^* \cdot (a_0 - a_1 i_t^*) \quad (10)$$

Right after the attack, we have:



As the demand for real balances falls, if  $P$  is not to jump,  $M$  has to fall by a discrete amount.

Figure 1: Flood and Garber (1984)

$$M^L = \bar{S} \cdot P_t^* \cdot \left( a_0 - a_1 \left( i_t^* + \frac{\dot{S}}{\bar{S}} \right) \right) \quad (11)$$

Subtracting (11) from (10), we get:

$$\Delta M = M^H - M^L = \bar{S} \cdot P_t^* \cdot a_1 \cdot \frac{\dot{S}}{\bar{S}}$$

As  $M_t = R_t + D_t$ ,  $\Delta M = \Delta R + \Delta D$ . Domestic credit is growing continuously. Therefore,  $D_t$  is the same right before and right after the devaluation ( $\Delta D = 0$ ). So,  $\Delta M = \Delta R$ :

$$\Delta R = \bar{S} \cdot P_t^* \cdot a_1 \cdot \frac{\dot{S}}{\bar{S}} = \bar{S} \cdot P_t^* \cdot a_1 \cdot \frac{\dot{P}}{P}$$

Interpreting the above equations: the fall in reserves corresponds to the fall in the demand for money. The fall in the demand for money is due to inflation post-devaluation. Inflation occurs because the Central Bank has run out of reserves (so cannot finance the fiscal authority by selling reserves anymore) and thus starts to finance the fiscal authority via inflation.

#### Some take home points

- Inconsistency between domestic policy and exchange rate policy leads to speculative attacks. Increases in  $D$  lead either to decreases in  $R$  (reserves dwindle) or to increases in  $M$  (monetary expansion, that leads to inflation). Loss of reserves can't go forever (stock of reserves available to Central Banks is finite). At some point, increases in  $D$  lead to increases in  $M$ .
- A simple demand for money relation, arbitrage in all markets (PPP, IRP) and the increase in domestic credit lead to agents massively sell domestic currency and force the abandonment of the peg.
- A massive speculative attack is not incompatible with rational agents.
- What to do about speculative attacks? The model seems to say: *“don't shoot the messenger!”*
- The model predicts that crises are predictable, anticipated.
- Inflation follows the currency crises.

However...

European Exchange Rate Mechanism, 1993: the bark in the dark that was not heard (Obstfeld, 1996).

## 1.2 The second generation models of currency crises

The weak links between changes in economic variables and speculative attacks in some recent episodes (e.g., the ERM crises in 1992-3 and the contagion of 1997-8) have stimulated the idea that bad fundamentals may be a pre-condition for a crisis, but its occurrence and timing are somewhat random events. The so called second generation models of currency crisis formalize this view. This literature points out that if fundamentals are not good enough, the optimal strategy for an agent in a currency crisis game depends on expectations: if everybody is expected to attack the currency, it is optimal to attack it, but if everybody is expected to refrain from doing so, then not attacking is the optimal choice. Those models present multiple equilibria. Sudden and exogenous shifts on expectations may trigger a crisis.

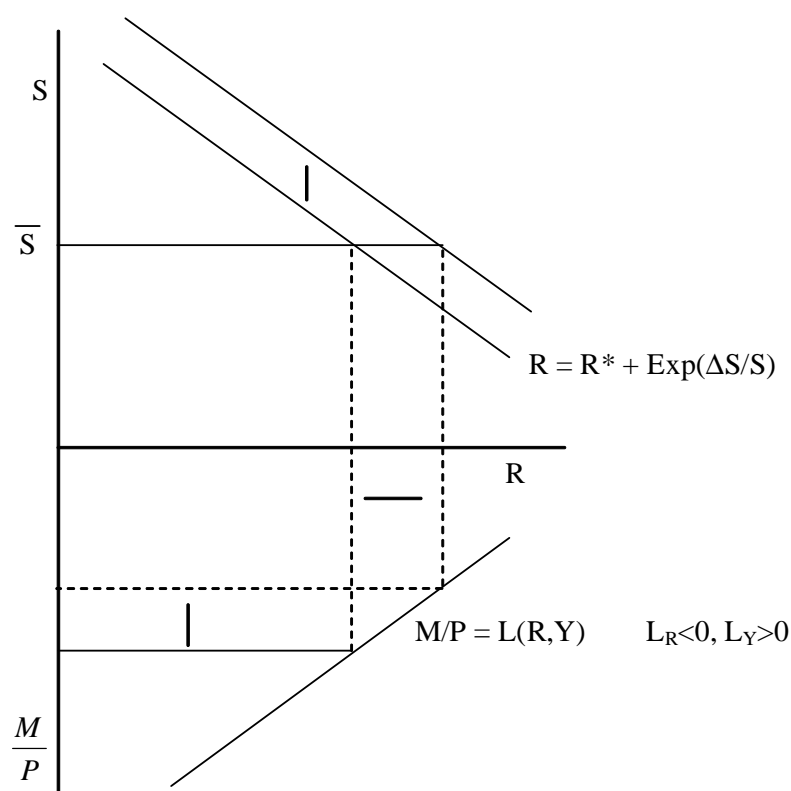


Figure 2: Change in expectations

A second generation model of currency needs:

1. A reason why the government want to abandon its fixed exchange rate regime,
2. A reason why the government want to keep its fixed exchange rate regime,

3. Cost of defending the fixed exchange rate regime must be increasing in expectations of devaluation.

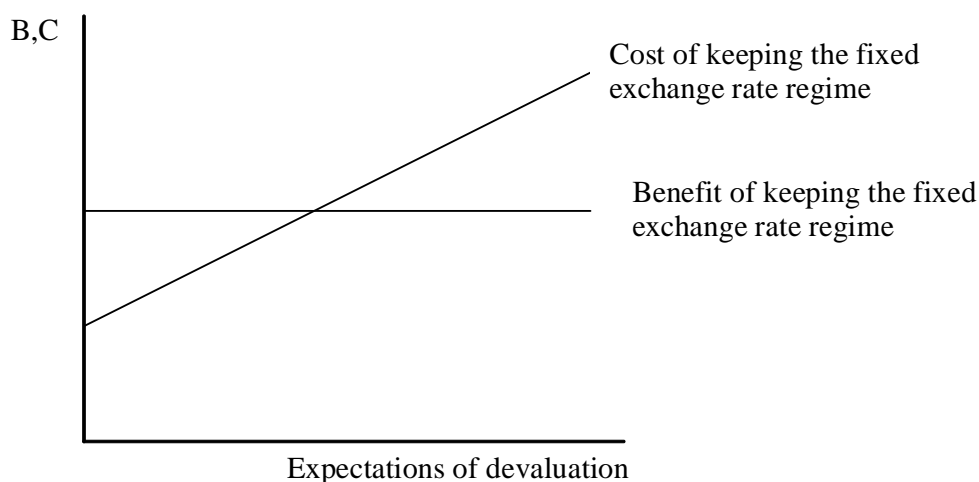


Figure 3: Government's decision on the exchange rate regime

What are the benefits of keeping the fixed exchange rate regime?

- Removing volatility of the exchange rate regime is good for trade, investment.
- Nominal anchor - inflation.
- Reputation.

What are the costs of keeping the fixed exchange rate regime? (See Obstfeld, 1996)

- Increases in the interest rate may slow down economy, increase unemployment.
- Distribution effects: hikes in the interest rate make mortgages more expensive, bond holders wealthier, indebted companies poorer.
- Banks may suffer when interest rates increases (we will discuss this issue further in a couple of weeks).
- If a country is highly indebted, its fiscal burden increases if expectations of a devaluation push up interest rates.
- Government may want to inflate away its debt.

Why would the costs of keeping the fixed exchange rate regime be increasing:



- The higher is the expected devaluation, the higher is the hike in the interest rates.
- Seeing from a different perspective, if one expects a currency to depreciate, he/she will sell it (or short it). The pressure for devaluation is proportional to this amount sold (or short) as the government will have to buy it or to increase incentives (interest rates) for others to hold it.

### 1.2.1 Jeanne (1997)

Jeanne (1997) develops a simple second generation model of currency crises (that he uses to execute a likelihood ratio test for the existence of multiple equilibria in the French Franc crisis).

#### Setup of the model

- The currency is pegged at a fixed rate with a foreign currency.
- Policymaker can defend the peg (possibly at some cost) or abandon it. He may be in:
  - “soft” mood, with probability  $\mu$ : maintains the peg if the net benefit of doing so is positive.
  - “tough” mood, with probability  $1 - \mu$ : maintains the peg whatever is the circumstance.

He needs this changes in moods to fit better the data, the model would go through without it.

- Net benefit of maintaining the peg:

$$B_t = b_t - \alpha \cdot \pi_{t-1}$$

where  $b_t$  is the gross benefit of the fixed peg and  $\pi_{t-1}$  is the probability evaluated by the private sector that the peg will be abandoned next period. So,  $b_t$  depends on economic fundamentals but  $B_t$  also depends on expectations. We assume that:

$$b_t = \phi_{t-1} + \epsilon_t$$

where  $\phi_{t-1} = E_t b_{t+1}$  and  $\epsilon_t$  is an error term.

## Equilibria

- The devaluation probability must be equal to the probability that the government is soft and the net benefit of maintaining the peg is negative, that is:

$$\begin{aligned}
 \pi_t &= \mu \cdot \Pr(B_{t+1} < 0) \\
 &= \mu \cdot \Pr(\epsilon_{t+1} < \alpha \cdot \pi_t - \phi_t) \\
 &= \mu \cdot F(\alpha \cdot \pi_t - \phi_t)
 \end{aligned}$$

So  $\pi_t$  appears at both sides of the equation. There may be multiple equilibria or not, depending on  $f(0)$ .

There are multiple equilibria if:

1. the parameters satisfy  $\mu \alpha f(0) > 1$ ;
2. the fundamental lies in the multiple-equilibrium interval;

Pictures in the paper show that if  $F()$  reacts strongly enough to  $\pi_t$ , there is a region with multiple equilibria.

### 1.2.2 A game theoretical representation: Obstfeld (1996)

As government's decision depends on how many agents attack the currency, self-fulfilling crises may occur. Everybody expects that the peg will be abandoned, so everybody attacks the currency. And the peg is abandoned because everybody attacked the currency. This kind of circular logic is characteristic of the second generation models of currency crises.

A simple example from Obstfeld (EER 1996) helps to clarify this point: a government that wants to fix its currency and two private holders of domestic currency who can sell it (attack the currency) or hold it (not attack). The government has  $R$  reserves to defend the peg. Each trader has domestic money resources of 6 which can be sold for reserves. To sell and take a position against the government, there is a cost of 1 (assumed to be irrespective of the amount sold, but that is not important for the results). In the event of giving up its peg, the government devalues by 50 percent (so, the traders get 1/2 unit of money for each unit they bought in the event of a successful attack).

The "high-reserve" game:  $R = 20$

		Trader 2	
		A	N
Trader 1	A	-1, -1	-1, 0
	N	0, -1	0, 0

The “low-reserve” game:  $R = 6$

		Trader 2	
		A	N
Trader 1	A	$1/2, 1/2$	$2, 0$
	N	$0, 2$	$0, 0$

The “intermediate-reserve” game:  $R = 10$

		Trader 2	
		A	N
Trader 1	A	$3/2, 3/2$	$-1, 0$
	N	$0, -1$	$0, 0$

**Take home points**

- There are self-fulfilling crises in a model with very rational investors.
- *Sunspots*, events completely disconnected from the economy, may change expectations and trigger a currency crisis.
- Fixed exchange rate regimes that would work well in the absence of the speculative attack may fall without major fundamental imbalances.
- There are *strategic complementarities* between agents’ actions: incentives for an agent to attack the currency are increasing in the share of agents that choose to attack the currency.
- Crises are not fully predictable.
- The game with multiple equilibria works not only for crises, but for many other things. For example, your willingness to go to a party may be an increasing function of the others’ decisions. So there might be 2 equilibria (either many people go to the party or noone goes).
- Here there are complementarities because the actions of the government depend on what everyone does, but strategic complementarities could arise for other reasons. Example: suppose you work for an asset management company. If you lost money in Asia and all your peers did the same, or if you all profit from investing there, this is business as usual for you. If you are the only one that lost money there, your job may be in danger. On the other hand, if you are the only one that got it right, you may get an extra bonus. However, you may prefer the former safe choice to the latter risky lottery, so it may be rational for you to try to do what others are doing.

However. . .

- Assumption that agents know what others are doing in equilibrium is very strong (is it?)
- Expectations are given exogenously in the model. How would you form your own expectations about what other players would do? Compare it with your own decision in the game played in class.
- A side point: The effect of an increase in foreign interest rates ( $R^*$ ) can also be seen at figure 2. If  $R^*$  increases, the interest-rate-parity curve shifts up, which forces the domestic economy to increase interest rates (if a devaluation is to be avoided). In 1994, the US Federal Reserve Bank sharply increased interest rates. Interest rates in Mexico had to follow. Clearly, that increases the cost of keeping the fixed exchange rate regime for the Mexican government. The currency crises occurred in Mexico in December 1994.

## 2 Expectations and coordination

### 2.1 Expectations and higher order beliefs

#### 2.1.1 The common knowledge assumption

If an event (say  $\theta > 0$ ) is common knowledge, then everybody knows it, everybody knows that everybody knows it, everybody knows that everybody knows that everybody knows it, everybody knows that everybody knows that everybody knows that everybody knows it, and so on.

It is not easy to see why that would be different from the simple “everybody knows it”. The example in Geanakoplos (1992, page 54) illustrates the strenght of the common knowledge hypothesis.

#### 2.1.2 Beauty contests

Consider the following game:

		Player 2	
		A	N
Player 1	A	$\theta, \theta$	$\theta - 1, 0$
	N	$0, \theta - 1$	$0, 0$

If  $\theta \in (0, 1)$ , the game has 2 Nash Equilibria:  $(A, A)$  and  $(N, N)$ . What would you choose?

The optimal choice depends on the probability attached to the other agent choosing  $A$ . Let's denote this probability by  $p$ , that is,  $p = \Pr(s_2 = A)$ . Then, my payoff from choosing  $A$  is:

$$\begin{aligned}\pi(s_1 = A) &= p.\theta + (1 - p).(\theta - 1) \\ &= p + \theta - 1\end{aligned}$$

So,  $A$  is the optimal choice if:

$$p > 1 - \theta$$

A natural question is: what does  $p$  depend on? What is the probability that the other player will choose  $A$ ? Naturally, that depends on the probability player 2 assigns for player 1 choosing  $A$ ...

Keynes, in the “General Theory of Employment, Interest and Money” (1936) wrote that: “...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the

prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgement, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.”

In a Nash equilibrium, an agent *knows* which action the other players will be choosing. So, the above mentioned reasoning is not incorporated in the second generation models of currency crises.

### 2.1.3 The model with incomplete information

Carlsson and Van Damme (Econometrica, 1993) show that multiplicity of equilibrium comes from two modelling assumptions:

- In equilibrium, agents know what others will do;
- All information is common knowledge.

What happens when we remove those assumptions and agents are uncertain about what others will do? An agent has to estimate the likelihood of other players attacking the currency. So, they try to assess the others' information — what do they know? what are they expecting? Like in the beauty contest example, an agent is guessing what the others guess that she knows.

Consider that agents are playing the same game they played before:

		Player 2	
		A	N
Player 1	A	$\theta, \theta$	$\theta - 1, 0$
	N	$0, \theta - 1$	$0, 0$

However,  $\theta$  is not observed. The *only* information agents have about  $\theta$  is a noisy signal  $x$ . Formally, the prior on  $\theta$  is uniformly distributed in the real line (or think of a normal distribution with  $\sigma \rightarrow \infty$ ) For each agent  $i$ :

$$x_i = \theta + \epsilon_i$$

The error term,  $\epsilon_i$ , is independent across agents. The striking result first proved by Carlsson and Van Damme is that the above game has a unique equilibrium *even if the variance of the error term is very small* — for example, even if  $\epsilon_i$  is distributed uniformly between  $-0.01$  and  $0.01$ , that is:  $\epsilon_i \sim U(-0.01, 0.01)$ .

The key here is that it is never common knowledge that  $\theta \in [0, 1]$ . For example, suppose agent 1 gets a signal  $x_1 = 0.40$ . Then:

- Agent 1 knows that  $\theta \in [0.39, 0.41]$ ,
- so agent 1 knows that  $x_2 \in [0.38, 0.42]$ ,
- so agent 1 knows that agent 2 knows that  $\theta \in [0.37, 0.43]$ ,
- so agent 1 knows that agent 2 knows that  $x_1 \in [0.36, 0.44]$ ,
- so agent 1 knows that agent 2 knows that agent 1 knows that  $\theta \in [0.35, 0.45]$ ,
- so agent 1 knows that agent 2 knows that agent 1 knows that  $x_2 \in [0.34, 0.46]$ ,
- so agent 1 knows that agent 2 knows that agent 1 knows that agent 2 knows that  $\theta \in [0.33, 0.47]$ , and so on.

An agent that gets a signal  $x_1 = 0.40$  thinks that perhaps  $\theta = 0.39$ , so  $x_2$  might be  $0.38$ , in which case agent 2 would think it is possible that  $\theta = 0.37$  and  $x_1 = 0.36$ . So agent 1 considers it possible that agent 2 thinks that  $x_1$  is as low as  $0.36$  or as high as  $0.44$ . Likewise, if  $x_1 = 0.36$ , agent 1 would consider it possible that agent 2 thinks that  $x_2$  is as low as  $0.32$ . Combining both statements means that agent 1 with  $x_1 = 0.40$  thinks that agent 2 might think that agent 1 might think that agent 2 might think that  $x_1$  is as low as  $0.32$ .

OK, but why does it matter?

#### 2.1.4 Unique equilibrium: an intuition

Suppose that  $\epsilon_i \sim N(0, \sigma)$ .

From the point of view of agent 1, that got signal  $x_1$

- $\theta \sim N(x_1, \sigma)$ .
- $x_2 \sim N(x_1, \sqrt{2}\sigma)$ . (Why? From the point of view of agent 1,  $\theta \sim N(x_1, \sigma)$ . and  $x_2 \sim N(\theta, \sigma)$ , so...)

The expected payoff from choosing  $A$  for an agent that got signal  $x_1$  is:

$$E(\pi(s_1 = A)) = p + x_1 - 1$$

### Dominant regions

- If  $x < 0$ ,  $E(\pi(s_1 = A)) < p - 1 < 0$ . Regardless of what player 2 is choosing, player 1 is better off by choosing  $N$ .
- If  $x > 1$ ,  $E(\pi(s_1 = A)) > p > 0$ . Regardless of what player 2 is choosing, player 1 is better off by choosing  $A$ .

Now, what if  $0 < x < 1$ ? Suppose for example that  $\sigma = 0.1$ .

If  $x_1 = 0.1$ , what is the probability that agent 2 will play  $A$ ?

$$\begin{aligned}\Pr(x_2 < 0) &= \Pr\left(\frac{x_2 - 0.1}{0.1\sqrt{2}} < \frac{-0.1}{0.1\sqrt{2}}\right) \\ &= \Phi\left(\frac{1}{\sqrt{2}}\right) = 0.24\end{aligned}$$

where  $\Phi$  is the standard normal cumulative distribution function. So, with probability 0.24, agent 2 got a signal smaller than 0 and will play  $N$ . If agent 2's signal is positive, we don't know what he will do, but we can say that:

$$p < 0.76$$

So, agent 1's expected payoff of attacking is:

$$E(\pi(s_1 = A)) < 0.76 + 0.1 - 1 = -0.14$$

so, agent 1 will *not attack*.

Using a similar argument, if  $x_1 = 0.9$ , then:

$$\begin{aligned}\Pr(x_2 > 1) &= \Pr\left(\frac{x_2 - 0.9}{0.1\sqrt{2}} > \frac{1 - 0.9}{0.1\sqrt{2}}\right) \\ &= 1 - \Phi\left(\frac{1}{\sqrt{2}}\right) = 0.24\end{aligned}$$

So, with probability 0.24, agent 2 got a signal bigger than 1 and will play  $A$ . If agent 2's signal is smaller than 1, we don't know what he will do, but we can say that:

$$p > 0.24$$

So, agent 1's expected payoff of attacking is:

$$E(\pi(s_1 = A)) > 0.26 + 0.9 - 1 = 0.14$$

so, agent 1 will *attack*.



Now, suppose that  $x_1 = 0.2$ . Then the probability that agent 1 gets a signal smaller than 0 is just 0.08. If all we can say is that  $p < 0.92$ , then we cannot say anything about agent 1's optimal decision in this case because

$$\begin{aligned} E(\pi(s_1 = A)) &= p + x_1 - 1 \Rightarrow \\ E(\pi(s_1 = A)) &< 0.92 + 0.2 - 1 = 0.12 \end{aligned}$$

that is, from this calculation, we can't tell whether  $E(\pi(s_1 = A))$  is positive or negative. How can agent 1 decide in this case?

Agent 2 is also rational and is doing the same calculations agent 1 is doing. Therefore, agent 1 knows that if  $x_2 = 0.10$ , agent 2 will not attack. Why is that? If agent 2 got signal  $x_2 = 0.10$ , he will consider that the probability that agent 1 got a negative signal is 0.24 and will not attack. Agent 1 knows that. And given that his signal  $x_1 = 0.20$ , he knows that agent 2 will have got a signal  $x_2 \leq 0.10$  with probability 0.24 because:

$$\Pr(x_2 < 0.1) = \Pr\left(\frac{x_2 - 0.2}{0.1\sqrt{2}} < \frac{0.1 - 0.2}{0.1\sqrt{2}}\right)$$

So, he knows that  $p < 0.76$  and therefore, agent 1's expected payoff of attacking is:

$$E(\pi(s_1 = A)) < 0.76 + 0.2 - 1 = -0.04$$

$E(\pi(s_1 = A))$  is negative! If  $x_1 = 0.2$ , agent 1 will choose  $N$ .

Now, suppose that we know that players 1 and 2 choose  $N$  if  $x \leq x_L$ , for some  $x_L$ .

Suppose  $x_1 = x_L + \eta$ , where  $\eta$  is a positive and very small constant,  $\eta \ll \sigma$ .

Then:

$$\begin{aligned} \Pr(x_2 \leq x_L) &= \Pr\left(\frac{x_2 - x_1}{\sqrt{2}\sigma} \leq \frac{x_L - x_1}{\sqrt{2}\sigma}\right) \\ &= \Pr\left(z \leq \frac{\eta}{\sqrt{2}\sigma}\right) \approx 0.5 \end{aligned}$$

So,

$$\begin{aligned} p &= \Pr(s_2 = A) \leq 1 - \Pr(x_2 \geq x_L) \\ &\leq 0.5 \end{aligned}$$

Then,

$$\begin{aligned} E(\pi(s_1 = A)) &= p + x_1 - 1 \\ &\leq x_1 - 0.5 \end{aligned}$$

Therefore, we can iteratively delete  $s = A$  whenever  $x_1 < 0.5$ .

At the other side, suppose that we know that players 1 and 2 choose  $A$  if  $x \geq x_H$ , for some  $x_H$ .

Suppose  $x_1 = x_H - \eta$ , where  $\eta$  is a positive and very small constant,  $\eta \ll \sigma$ .

Then:

$$\begin{aligned} \Pr(x_2 \geq x_H) &= \Pr\left(\frac{x_2 - x_1}{\sqrt{2}\sigma} \geq \frac{x_H - x_1}{\sqrt{2}\sigma}\right) \\ &= \Pr\left(z \geq \frac{\eta}{\sqrt{2}\sigma}\right) \approx 0.5 \end{aligned}$$

So,

$$\begin{aligned} p &= \Pr(s_2 = A) \geq \Pr(x_2 \geq x_H) \\ &\geq 0.5 \end{aligned}$$

Then,

$$\begin{aligned} E(\pi(s_1 = A)) &= p + x_1 - 1 \\ &\geq x_1 - 0.5 \end{aligned}$$

Therefore, we can iteratively delete  $s = N$  whenever  $x_1 > 0.5$ .

So, the unique equilibrium that survives strategically elimination of strictly dominated strategies is:

- $s_i = A$  if  $x_i > 0.5$
- $s_i = N$  if  $x_i < 0.5$

The argument works in the same way even if the support of  $\varepsilon$  is bounded, for example, even if  $\varepsilon_i \sim U(-0.01, 0.01)$ . Note the higher order beliefs in action: if you get a low signal (say  $x = 0.05$ ) you will end up playing  $N$  even though you know that  $\theta$  is positive and you know that the other player knows that  $\theta$  is positive. That is because he may think that you may think that he may think that ..... that  $\theta$  is negative. Although agents in this case know that  $\theta > 0$ , so the  $(A, A)$  equilibrium would yield positive payoffs for them, that is not *common knowledge*.

### 2.1.5 Morris and Shin (1998)

Morris and Shin (1998), building on Carlsson and Van Damme (Econometrica, 1993), endogenize expectations in a model of currency attacks. Here I present a simplified version.

Consider an economy with a continuum of players and denote by  $l$  the proportion of players that choose to attack. The government has a constant benefit for holding the peg and a cost that depends negatively on fundamentals ( $\theta$ ) and positively on the proportion of agents that choose to attack ( $l$ ). We will consider that the cost will exceed the benefit if  $l - \theta > 0$ . So, the government abandons the peg if  $l > \theta$ .

The information structure is as before ( $x_i = \theta + \epsilon_i$ ). An agent chooses between ‘attack’ ( $A$ ) and ‘not attack’ ( $N$ ). If the agent chooses  $N$ , she does not win or lose anything, her payoff is 0 regardless of what others do. The cost of attacking is  $t$ . If she attacks and there is a devaluation, she gets 1. So, if she chooses  $A$ , her payoff is  $1 - t$  if there is a devaluation and  $-t$  otherwise. Suppose that  $\epsilon_i \sim N(0, \sigma)$ .

Here, if  $\theta$  is negative, the government abandons the peg regardless of what agents do and if  $\theta > 1$ , the peg is kept even if all agents decide to attack. The agent will attack the currency if she perceives that fundamentals are weak, that is, only if she thinks that  $\theta$  is small.

**The model with common knowledge of fundamentals** Suppose for a while that all agents observe  $\theta$ . Then, the model is a second generation model.

Suppose everyone attacks the currency ( $l = 1$ ). Then the government abandons the peg if  $\theta < 1$ . In this case, it is optimal for an agent to attack the currency if  $\theta < 1$ .

Suppose that no one attacks the currency ( $l = 0$ ). Then the government abandons the peg if  $\theta < 0$ . In this case, it is optimal for an agent not to attack the currency if  $\theta > 0$ .

Thus, if  $\theta < 0$ , fundamentals are too weak, everybody attacks the currency and the government leaves the peg. If  $\theta > 1$ , nobody attacks and the peg survives. But if  $0 < \theta < 1$ , both equilibria exist.

**The model with imperfect information about  $\theta$**  Now, let’s return to the case when  $\theta$  is not observed and agents have just imperfect information about it.

Morris and Shin (1998) show that there is a unique equilibrium, characterized by two thresholds:

1. an agent attacks only if her signal  $x_i$  is smaller than  $x^*$ .
2. the government abandons the exchange rate peg only if  $\theta < \theta^*$ .

2 conditions pin down the equilibrium:

1. An agent who gets signal  $x^*$  is indifferent between attacking or not attacking.

2. When  $\theta = \theta^*$ , the fraction of agents that attack the currency is just enough to make the government abandon the peg.

An agent that gets signal  $x^*$  asks himself: what is the probability that the attack will succeed?

- The question is: what is the probability that  $\theta < \theta^*$ ?
- From that agent's point of view,  $\theta \sim N(x^*, \sigma)$ . Saying differently,  $\theta = x^* - \epsilon_i$ .
- So,  $\Pr(\theta < \theta^*) = \Pr(x^* - \epsilon_i < \theta^*) = \Pr(-\epsilon_i < \theta^* - x^*) = \Pr(\epsilon_i < \theta^* - x^*)$ . Thus:

$$\Pr(\theta < \theta^*) = \Phi\left(\frac{\theta^* - x^*}{\sigma}\right)$$

where  $\Phi$  is the cumulative standard normal distribution.

So, The expected payoff of attacking is:

$$\begin{aligned} E(\text{pay}A) &= (1-t) \cdot \Phi\left(\frac{\theta^* - x^*}{\sigma}\right) - t(1 - \Phi\left(\frac{\theta^* - x^*}{\sigma}\right)) \\ &= \Phi\left(\frac{\theta^* - x^*}{\sigma}\right) - t \end{aligned}$$

The agent is indifferent when:

$$\Phi\left(\frac{\theta^* - x^*}{\sigma}\right) - t = 0 \tag{12}$$

The second equilibrium condition: when  $\theta = \theta^*$ , the fraction of agents that attack the currency is just enough to make the government abandon the peg. But when  $\theta = \theta^*$ , what is the proportion of agents that choose to attack?

- As we have many agents, the question is: what is the proportion of agents that get a signal  $x$  such that  $x < x^*$ ?
- If  $\theta = \theta^*$ ,  $x = \theta^* + \epsilon_i$ .
- So,  $\Pr(x < x^*) = \Pr(\theta^* + \epsilon_i < x^*) = \Pr(\epsilon_i < x^* - \theta^*)$ . Thus:

$$\Pr(x < x^*) = \Phi\left(\frac{x^* - \theta^*}{\sigma}\right)$$

When  $\theta = \theta^*$ , the cost and the benefit of government keeping the peg are the same. So:

$$\theta = l \Rightarrow \theta^* = \Phi\left(\frac{x^* - \theta^*}{\sigma}\right) \tag{13}$$

Remember that:

$$\Phi\left(\frac{\theta^* - x^*}{\sigma}\right) = 1 - \Phi\left(\frac{x^* - \theta^*}{\sigma}\right)$$

So, the two equilibrium conditions (equations 12 and 13) yield:

$$1 - \theta^* - t = 0$$

So:

$$\theta^* = 1 - t$$

### A few take home points

- If agents are trying to guess what others are trying to do in a currency crises situation, expectations are crucial for the final outcome but are not disconnected from economic fundamentals.
- In this simple model, expectations depend only on prices ( $t$ ) and fundamentals ( $\theta$ ). However, more elaborated models using this technique are able to show other interesting features of expectations (for example, the effect of public information in crises).

#### 2.1.6 The effect of public information

Let's look at the magnifying effect of public information (from Morris and Shin, 2003).

Consider again the following game:

		Player 2	
		A	N
Player 1	A	$\theta, \theta$	$\theta - 1, 0$
	N	$0, \theta - 1$	$0, 0$

Suppose that agents do not observe  $\theta$ . The information agents have about  $\theta$  is a noisy private signal  $x_i$  and a public signal  $y$ . For each agent  $i \in \{1, 2\}$ :

$$x_i = \theta + \varepsilon_i, \varepsilon_i \sim N(0, \sigma^2)$$

and  $y$  is the same for all agents:

$$y = \theta + \eta, \eta \sim N(0, \tau^2)$$

The private signal is the result of your own analysis of the economy. The public signal is in the first page of the Financial Times and you are sure that everybody will read it.

Agent  $i$ 's posterior expectation of  $\theta$  is normally distributed with mean:

$$\bar{\theta}_i = E(\theta|x_i, y) = \frac{\left(\frac{1}{\tau^2}\right) y + \left(\frac{1}{\sigma^2}\right) x_i}{\left(\frac{1}{\tau^2}\right) + \left(\frac{1}{\sigma^2}\right)} = \frac{\sigma^2 y + \tau^2 x_i}{\sigma^2 + \tau^2}$$

and variance

$$\frac{1}{\left(\frac{1}{\tau^2}\right) + \left(\frac{1}{\sigma^2}\right)} = \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}$$

Moreover, from the point of view of agent 1, the signal received by agent 2 is:

$$x_2 = \bar{\theta}_1 + \varepsilon_2$$

Since  $E(\varepsilon_2) = 0$ ,  $E_1(x_2) = \bar{\theta}_1$ . The variance of agent 1's estimator of  $x_2$  is:

$$Var(\bar{\theta}_1) + Var(\varepsilon_2) = \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2} + \sigma^2 = \frac{2\sigma^2 \tau^2 + \sigma^4}{\sigma^2 + \tau^2}$$

**An intuition** Let's consider the case  $\sigma^2 = \tau^2$ . For concreteness, suppose  $\theta = 0.5$ ,  $x_1 = 0.4$ ,  $x_2 = 0.4$  and  $y = 0.6$ . Remember that in the case with only private information, agents would be indifferent when  $x_i = 0.5$ . The question is: what happens in this case?

From the point of view of agent 1:

- $E(\theta) = 0.5$
- $E(x_2) = 0.5$

Now, what does agent 1 expect about agent 2's expected  $x_1$ ?

- Agent 1 considers that agent 2 has 2 pieces of information:  $x_2$  and  $y$ .
- From the point of view of agent 1,  $E(x_2) = 0.5$  and  $E(y) = 0.6$ .
- So,  $E_1(E_2(x_1)) > 0.5$  (it is a (weighted) average between 0.5 and 0.6).

Notice that both agents think that  $\theta$  is 0.5 plus an error term. But both agents think that the other agent's estimate of  $\theta$  is bigger than 0.5.

In this case, agents would be indifferent if all their information were private. But both expect that the other will attack because the information in the front page of the Financial Times matters more for what I think than what you think.

**Back to the model** Consider agents follow a cut-off strategy: they will invest if their posterior  $\bar{\theta}$  is larger than  $\kappa$ , which is a function of parameters and the public signal.

Agent 1 knows agent 2 will play  $A$  if her posterior is larger than  $\kappa$ , or

$$\frac{\sigma^2 y + \tau^2 x_2}{\sigma^2 + \tau^2} > \kappa \Leftrightarrow x_2 > \frac{(\sigma^2 + \tau^2) \kappa - \sigma^2 y}{\tau^2} \Leftrightarrow x_2 > \kappa + \frac{\sigma^2}{\tau^2} (\kappa - y)$$

and given the distribution of probability of  $x_2$  (from the point of view of agent 1) the probability  $x_2$  is smaller than  $\kappa + (\sigma^2/\tau^2) (\kappa - y)$  is given by:

$$\Phi \left( \sqrt{\frac{\sigma^2 + \tau^2}{2\sigma^2\tau^2 + \sigma^4}} \left[ \kappa - \bar{\theta}_1 + \frac{\sigma^2}{\tau^2} (\kappa - y) \right] \right)$$

Hence agent 1's the expected payoff from investing is:

$$v(\bar{\theta}_1, \kappa) = \bar{\theta}_1 - \Phi \left( \sqrt{\frac{\sigma^2 + \tau^2}{2\sigma^2\tau^2 + \sigma^4}} \left[ \kappa - \bar{\theta}_1 + \frac{\sigma^2}{\tau^2} (\kappa - y) \right] \right)$$

In equilibrium, an agent with with posterior  $\bar{\theta}_i = \kappa$  has to be indifferent, so  $\kappa$  is given by:

$$v^*(\kappa) = v(\kappa, \kappa) = \kappa - \Phi(\sqrt{\gamma}(\kappa - y)) \quad (14)$$

where  $\gamma$  is given by

$$\gamma = \frac{\sigma^2}{\tau^4} \left( \frac{\sigma^2 + \tau^2}{\sigma^2 + 2\tau^2} \right)$$

The derivative of  $v^*$  with respect to  $\kappa$  is given by:

$$\frac{dv}{d\kappa} = 1 - \sqrt{\gamma} \phi(\sqrt{\gamma}[\kappa - y])$$

If  $\gamma$  is large enough, this derivative is negative for some values of  $y$  and we have multiple equilibria depending on the realization of the public signal. That is illustrated in Figure 3.4 of Morris and Shin (2003). Since  $\phi(\sqrt{\gamma}[\kappa - y]) \leq 1/\sqrt{2\pi}$ , there is a unique equilibrium in the model whenever  $\gamma \leq \sqrt{2\pi}$ .

**How does the public information affect the equilibrium?** Now, suppose  $\gamma \leq \sqrt{2\pi}$ , so there is a unique equilibrium. The equilibrium condition  $v^*(\kappa) = 0$  in (14) for  $\kappa = \bar{\theta}_i$  can be written as:

$$\frac{\sigma^2 y + \tau^2 x_i}{\sigma^2 + \tau^2} - \Phi \left( \sqrt{\gamma} \left( \frac{\sigma^2 y + \tau^2 x_i}{\sigma^2 + \tau^2} - y \right) \right) = 0$$

The question now is how much the private signal has to change to compensate for a change in the public signal and still leave agent 1 indifferent. The answer can be obtained by applying the implicit function theorem to the above equilibrium condition:

$$\frac{dx_i}{dy} = - \frac{\frac{\sigma^2}{\tau^2} + \sqrt{\gamma} \phi(\cdot)}{1 - \sqrt{\gamma} \phi(\cdot)} \quad (15)$$

We can also calculate how much the private signal would have to change to compensate for a change in the public signal if there was no strategic effect. Fix agent  $i$ 's posterior:

$$\bar{\theta}_i = \frac{\sigma^2 y + \tau^2 x_i}{\sigma^2 + \tau^2} = c$$

Then the implicit function theorem yields:

$$\frac{dx_i}{dy} = -\frac{\sigma^2}{\tau^2} \quad (16)$$

which is smaller (in absolute value) than the derivative in (15). We can define the publicity multiplier as the ratio between (15) and (16), which yields:

$$\begin{aligned} \zeta &= \frac{1 + \frac{\tau^2}{\sigma^2} \sqrt{\gamma} \phi(\cdot)}{1 - \sqrt{\gamma} \phi(\cdot)} \\ &= \frac{1 + \frac{1}{\sigma} \sqrt{\frac{\sigma^2 + \tau^2}{\sigma^2 + 2\tau^2}} \phi(\cdot)}{1 - \frac{\sigma}{\tau^2} \sqrt{\frac{\sigma^2 + \tau^2}{\sigma^2 + 2\tau^2}} \phi(\cdot)} \end{aligned}$$

which is larger than 1 and decreasing in  $\tau$ . In case  $\tau \rightarrow \infty$ ,  $\theta \rightarrow 0$ . Hence the derivative in (15) is zero (as one would expect), but the multiplier is still larger than 1. As the precision of the public information increases ( $\tau$  decreases), the multiplier gets larger, public information has a stronger effect on the equilibrium.

### 2.1.7 Discussion about contagion

There is large evidence that a crisis in a country makes other countries more subject to crises. The Mexican crisis in the end of 1994 had large impacts in other Latin American countries, especially in Argentina. The Asian crisis started in Thailand in July/1997 and was spread to many other countries in Southern Asia. Latin American countries were also affected (the Asian Flu). The Russian crisis in 1998 also had strong impacts in other developing countries like Brazil and Mexico (the Russian virus). The Brazilian devaluation in 1999 let Argentina in a more fragile position.

What are the possible linkages between countries? Trade links or other fundamental links are plausible candidates, but one would be hard pressed to find strong commercial links between Thailand and Brazil. Changes in “first world” prices might have effects as well (for example, the strong impacts in Latin America of changes in US interest rates and the effects of the devaluation of the Yen with respect to the dollar in 1995-1996). What else?

A crisis in one country might generate information about the way the economy works that is relevant to other countries – and there is where the magnifying effects of public



information come in. A crisis in Thailand can be interpreted as a public signal, everyone knows about it. Even if we think that provides little information about Brazil, we know everyone else has got this information, and if coordination is important, that affects behaviour.

The literature also discusses a different channel, where contagion is caused by wealth effects: investors that lost money in a country need to withdraw from some other. Using the global games methodology, Goldstein and Pauzner (2004) present a model of contagion caused by wealth effects. And now it is advertising time again: Guimaraes and Morris (2007) extend the model in Morris and Shin (1998) in order to account for risk, wealth and portfolio effects.

## 2.2 Rational herd behavior

If my payoff depends on your actions, we may choose the same action due to the strategic complementarities discussed above. But conformity of behavior may arise because of an informational cascade. One agent observes actions of others and learns from it. That is called herd behavior or informational cascades

### 2.2.1 Bikhchandani, Hirshleifer and Welch (1992)

Here I go through the model at section 2 of BHW with a slight modification: BHW assume that if the agent is indifferent, she will adopt either action with probability 50%. I will assume that if the agent is indifferent, he will follow its own signal. It makes no difference in the general message but simplifies the algebra a bit.

Suppose agent 1 receives signal  $H$  (without loss of generality). Then, from his point of view:

$$\begin{aligned}\Pr(V = 1|H) &= p \\ \Pr(V = 0|H) &= 1 - p\end{aligned}$$

Since  $p > 0.5$ , he chooses to *adopt*. Thus, the following players know he has got signal  $H$ .

Suppose agent 2 receives signal  $H$ . From her point of view:<sup>1</sup>

$$\Pr((V = 1)|(H, H)) = \frac{\Pr((V = 1) \cap (H, H))}{\Pr((H, H) \cap (V = 0)) + \Pr((H, H) \cap (V = 1))}$$

So:

$$\Pr((V = 1)|(H, H)) = \frac{\frac{1}{2}p^2}{\frac{1}{2}p^2 + \frac{1}{2}(1-p)^2}$$

Similarly,

$$\Pr((V = 0)|(H, H)) = \frac{\frac{1}{2}(1-p)^2}{\frac{1}{2}p^2 + \frac{1}{2}(1-p)^2}$$

And, as  $p > \frac{1}{2}$ , she chooses to *adopt*. If she receives signal  $L$ , then, from her point of view:

$$\Pr((V = 1)|(H, L)) = \frac{\frac{1}{2}p(1-p)}{\frac{1}{2}p(1-p) + \frac{1}{2}p(1-p)} = \frac{1}{2}$$

$$\Pr((V = 0)|(H, L)) = \frac{\frac{1}{2}p(1-p)}{\frac{1}{2}p(1-p) + \frac{1}{2}p(1-p)} = \frac{1}{2}$$

She is indifferent. So, according to my tie-breaking convention, she follows her own signal and decides to *reject* — in the paper, due to their tie-breaking rules, she takes either action with 50% probability.

Now, consider agent 3. If agents 1 and 2 have taken different actions. Agent 3 knows that they have got different signals. Thus, before agent 3 sees her own signal, she thinks that:  $\Pr(V = 0) = \Pr(V = 1) = \frac{1}{2}$ . So, the game is as in the beginning.

What if agent 1 and 2 have both decided to *adopt*? Agent 3 knows that both have received signal  $H$ . If she also receives signal  $H$ , she will also decide to adopt, as from her point of view,  $\Pr(V = 1) > \Pr(V = 0)$  — is it clear for you without doing the calculations? But what if she receives signal  $L$ ? Then, she will think: “3 signals, 2  $H$  and 1  $L$ . Which is more likely:  $(V = 0)$  or  $(V = 1)$ ?”

$$\Pr((V = 1)|(H, H, L)) = \frac{\frac{1}{2}p^2(1-p)}{\frac{1}{2}p^2(1-p) + \frac{1}{2}p(1-p)^2}$$

$$\Pr((V = 0)|(H, H, L)) = \frac{\frac{1}{2}p(1-p)^2}{\frac{1}{2}p^2(1-p) + \frac{1}{2}p(1-p)^2}$$

---

<sup>1</sup>In general, given history  $h$ ,

$$\Pr(V = 1|h) = \frac{\Pr(V = 1 \cap h)}{\Pr(V = 1 \cap h) + \Pr(V = 0 \cap h)}$$

and with  $m$  realizations of signal  $H$  and  $n$  signals in total, that is:

$$\frac{\frac{1}{2}p^m(1-p)^{n-m}}{\frac{1}{2}p^m(1-p)^{n-m} + \frac{1}{2}(1-p)^m p^{n-m}}$$

Again, as  $p > \frac{1}{2}$ , she chooses to adopt, although she got signal  $L$ . The intuition is: more signals pointing to ( $V = 1$ ) makes ( $V = 1$ ) more likely.

What will agent 4 do? She knows that agent 3 would *adopt* anyway, so agent 3's action is uninformative. She knows that agents 1 and 2 have received signal  $H$ . Thus, she is in the same situation of agent 3 and will *adopt* regardless of her signal. Do I need to repeat this spiel for agent 5?

That characterizes an information cascade. Agents act regardless of their own information, not because they are crazy but precisely because they are rational — they extract information from others' actions instead of ignoring it.

A key (and interesting) feature of the equilibrium is that agents may observe many others moving before them, but only the actions of the first ones are informative. The others are just following the herd. So, the information they had is not passed ahead.

Discussion of the key assumptions:

- Action: 0 or 1. If there was a continuous set of actions and agents' optimal action depended on the probability they assigned to true value being 1, everybody would infer the signal received by an agent from her action. Thus, there will be no information cascade: individuals would always consider their own information and eventually the true state would be known.
- Exogenous order of movements (which can be relaxed, see Caplin and Leahy, 1994)
- My signal and your signal have (almost) the same weight in my decisions. What do you think of this assumption? Is it rational?

### 3 Banking crises

Banking crises are related to international financial crises for two reasons: because currency and debt crises are related to banking crises; and because some models used to study international financial crises are based on models of banking crises, especially Diamond and Dybvig (1983).

#### 3.1 Bank runs

The model of Diamond and Dybvig (1983) has become the standard model of bank runs in the literature. First, the model shows that banks issuing demand deposits can improve on a competitive market. However, that also creates an undesirable equilibrium (bank runs), in which banks that would have no problem if people just believed so might be victims of self-fulfilling prophecies. The presentation here also borrows from Diamond (2007).

##### 3.1.1 Diamond and Dybvig (1983)

Illiquidity of some assets in this model explain both the existence of banks and their vulnerability.

**Setup** There are 3 periods ( $t = 0, 1, 2$ ), one homogenous good, and measure-one continuum of agents.

Agents get an endowment of 1 unit at  $t = 0$  (and nothing else).

There are 2 technologies:

1. Storage, people can store goods from one period to the other. It is liquid and not publicly observable.
2. A productive technology: one unit invested at  $t = 0$  yields either 1 unit at  $t = 1$  or  $R$  units at  $t = 2$ . At  $t = 1$ , agents have to decide whether they wait until  $t = 2$  or whether they get 1 at  $t = 1$ .

At  $t = 0$ , consumers are all the same. But at  $t = 1$ , a consumer learns its own type, which might be:

1. Type 1: consumes at period one, utility is  $u(c_1)$ .
2. Type 2: consumes at period two, utility is  $u(c_2)$ .

The type is unobservable to others. To make the problem interesting,  $R > 1$ . Moreover the utility function is increasing and concave, the Inada conditions hold and the coefficient of risk aversion has to be large enough ( $-u''(c)c/u'(c) > 1$ ).

A fraction  $\theta \in (0, 1)$  of agents is type 1.

**Competitive markets can do nothing to improve allocation** An agent by itself can invest in the technology at  $t = 0$ . If the agent happens to be type 1, she consumes 1 at  $t = 1$ . If she happens to be type 2, she consumes  $1/R$  and gets  $u(1)$ , if she is type 1, she gets  $u(R)$ .

Competitive markets cannot improve upon that. Period 0 price of  $c_1$  has to be 1, and the price of  $c_2$  has to be  $R^{-1}$  at both  $t = 0$  and  $t = 1$ , otherwise there are arbitrage opportunities.

**If types were public observable...** A central planner that can observe everyone's type would choose  $c_1$  and  $c_2$  in order to maximize

$$\theta u(c_1) + (1 - \theta)u(c_2) \quad (17)$$

subject to the budget constraint

$$\theta c_1 + \frac{(1 - \theta)c_2}{R} = 1$$

hence (17) can be written as

$$\theta u(c_1) + (1 - \theta)u\left(\frac{R(1 - \theta c_1)}{1 - \theta}\right)$$

which yields

$$u'(c_1) = Ru'(c_2)$$

For example, if

$$u(c) = 1 - \frac{1}{c} \quad (18)$$

then

$$\frac{c_2}{c_1} = \sqrt{R}$$

which is different than what an agent can do by herself (and the competitive equilibrium). Here, the type one agent gets proportionally more and the type 2 agent gets less. There is risk sharing, more projects are interrupted at  $t = 1$  but that is good for the agents ex-ante. If risk aversion was low enough, that would not be true, because interrupting more projects at  $t = 1$  also means that the total available for consumption is smaller.

In equilibrium then,

$$c_1^* = \frac{\sqrt{R}}{1 - \theta + \theta\sqrt{R}} \quad , \quad c_2^* = \frac{R}{1 - \theta + \theta\sqrt{R}} > c_1^* \quad (19)$$

**In case of log utility...** The first order condition is

$$\frac{1}{c_1} = \frac{R}{c_2}$$

and using the budget constraint, we get:

$$c_1 = 1$$

which means agents do not want more risk sharing than what they can obtain by themselves.

**Demand deposit contracts** Now suppose there are competitive banks that can offer the following contract: agents deposit their money at  $t = 0$  at the bank. The bank invests in the new technology at  $t = 0$ . Agents can either get  $c_1^*$  at  $t = 1$  or whatever is left at  $t = 2$ . In order to pay agents at  $t = 1$ , the bank interrupts production using the productive technology.

Now suppose that the  $\theta$  type 1 agents choose to withdraw at  $t = 1$  and all the others wait until  $t = 2$ . Since  $c_1^*$  and  $c_2^*$  solve the central planner problem assuming  $\theta$  agents consume at  $t = 1$ , we know that the  $1 - \theta$  type 2 agents will get  $c_2^*$ . Since  $c_2^* > c_1^*$ , that is indeed the optimal strategy for type 2 agents, so the demand deposits implement the first best in this economy. That shows how banks can improve on a competitive market by issuing demand deposits.

But now suppose everyone decides to withdraw at  $t = 1$ . There will be no money left at  $t = 2$ . So for type 2 agents it is optimal to try to get their money at  $t = 1$  (it is a choice between the possibility of getting  $c_1^*$  and the certainty of getting zero). That means that if everyone expects all agents will withdraw early, withdrawing early is the optimal strategy. A bank run is thus a self-fulfilling prophecy.

Multiple equilibria arise here because the amount of resources left to  $t = 2$  is increasing on the number of agents that leave their money at the bank, and leaving the money at the bank pays off only if there are enough resources at the bank at  $t = 2$ . The fundamental frictions in the model are the illiquidity of the productive technology and the fact that types are unobservable.

**Example** Using the utility in (18), with  $\theta = 1/4$  and  $R = 2$  an agent's utility in the equilibrium without banks would be:

$$\begin{aligned} & \frac{1}{4}u(1) + \frac{3}{4}u(2) \\ &= \frac{1}{4} \left(1 - \frac{1}{2}\right) + \frac{3}{4}u \left(1 - \frac{1}{2}\right) \\ &= 0.375 \end{aligned}$$

Using the equilibrium condition in (19), we get that:

$$c_1^* = 1.28 \quad , \quad c_2^* = 1.813$$

and

$$\begin{aligned} & \frac{1}{4}u(1.28) + \frac{3}{4}u(1.813) \\ &= 0.391 > 0.375 \end{aligned}$$

How do we implement this equilibrium?

Instead of buying the risky asset, we can leave enough to pay the agents at  $t = 1$  in the illiquid asset. Since at  $t = 1$  we will need an amount equal to:

$$\frac{1}{4}1.28 = 0.32$$

we can leave 32% of the economy's resources for the type 1 agents and invest 68% of the resources in the illiquid technology. Then at  $t = 2$ , we will get, for each agent:

$$\frac{0.68 \times 2}{\frac{3}{4}} = 1.813$$

Hence the bank is not generating more units of consumption than the average agent in autarky, it is actually investing less in the illiquid technology. The benefits come from risk sharing.

### Extensions and policies

- When the illiquid asset yields  $\lambda < 1$  instead of 1 at  $t = 1$ , agents in autarky are worse off, but the bank is not, because it can buy the liquid asset (that yields 1 with certainty) for the type 1 consumers.
- Instead of consumers with risk aversion, liquidity could be demanded by entrepreneurs who may or may not get a profitable project at  $t = 1$  (see Diamond, 2007).

- Options considered in Diamond and Dybvig (1983) to deal with bank runs include the suspension of convertibility: the idea is that if I know not many agents will be able to withdraw at  $t = 1$  and there will be money left for type 2 agents, I don't have incentives to withdraw at  $t = 1$  (if I am a type 2 agent). The role of government policy in their paper is to prevent a bad equilibrium rather than moving an existing equilibrium.

**What else?** Using an approach similar to Morris and Shin (1998), Goldstein and Pauzner (2005) find a unique equilibrium in this model and analyze the effect of parameters on the equilibrium and the effect of different policies.

Chang and Velasco have a few papers with similar versions of a model of financial crises in emerging markets based on Diamond and Dybvig (1983), with banks in the domestic economy pooling resources and providing liquidity, but borrowing from abroad. See, e.g., Chang and Velasco (1998).

## 3.2 Banking crises and moral hazard

### 3.2.1 The moral hazard effect

The game: 'heads I win, tails the taxpayeres lose':

Suppose that there is a potential investment that will cost \$70 million up front. If all goes well, the project will yield \$90 million. That will occur with probability  $1/2$ . But with probability  $1/2$ , the return will only be \$30 million. The expected payoff then is  $(1/2 \times \$90m) + (1/2 \times \$30m) = \$60$  million. Ordinarily, this investment would never be made.

However, bailout guarantees change the result. Suppose that an investor is able to borrow the entire \$60 million because everyone (including him and the lenders) knows that the government will protect them if his project fails and he cannot repay. Then, from his point of view, he will make \$20 million ( $= \$90m - \$70m$ ) with probability half and walk away with nothing with probability half.

The solution seems to be simple: the government cannot bailout such projects. But that is not simple. Diaz-Alejandro (1985), studies the example of Chile in the 80's. "Good-bye financial repression, hello financial crash", that is what happened. Lots of bank regulations were removed with the aim of ending financial repression and the government of Chile had pledged not to bail-out banks if they crashed. However...



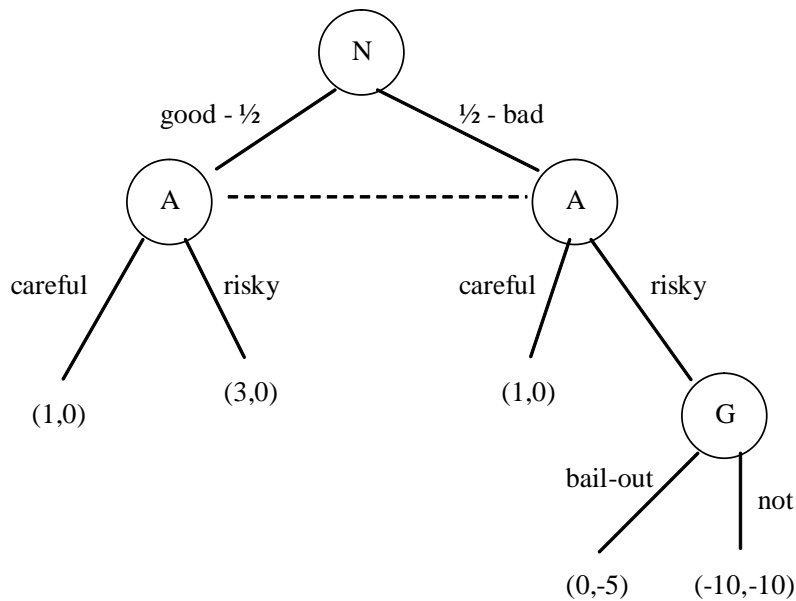


Figure 4: The moral hazard game

**The externality** A bank is a borrower and a lender. Its assets are bonds, loans, other financial assets. Its liabilities are loans, bonds and other financial instruments. If a bank crashes and it cannot pay its debts, its bankruptcy can lead to the failure of other banks and companies — it may be the first domino to fall — because its liabilities are other banks and companies assets.

So, bailing-out a failing bank is costly. But not bailing-out may be worse, due to the negative externalities spread to the rest of the economy.

**The credibility issue** Consider the game shown at figure 4.

Players:

- N: nature,
- A: agent (banks, big companies, lenders...),
- G: government.

When the government says: “I will not bail out insolvent banks/companies”, the government is saying: “I will play *not*”. If the government is indeed playing *not*, the agent gets payoff of 1 if he plays *careful* and expected payoff of -3.5 if he plays *risky*. Thus, the agent playing *careful* and the government playing *not* is a Nash equilibrium: nobody has any incentive to deviate.

However, this Nash equilibrium is not credible (it is not a Subgame Perfect Nash Equilibrium). If we get into a situation in which the government is called to action, choosing *not* yields -10, while choosing *bail-out* yields -5. None is great, but *bail-out* causes less damage. So, that is what the government chooses. Now, the agent knows this, the threat of not bailing-out is not *credible*. Thus, the agent gets payoff of 1 if he plays *careful* and expected payoff of 1.5 if he plays *risky*. Thus, the agent plays *risky* and, in the event of a bad state of nature, hello financial crash.

The key assumption in this game is that, once a banking crisis takes place, the payoff of not bailing out the banks is smaller than the payoff of helping them. Diaz Alejandro (1985) argues that is true in reality: promises to play *not* at the last node are not credible.

Policy implication: regulation, ex-ante actions are needed.

### 3.2.2 The Asian crisis of 1997

Many analysts have considered that moral hazard — the game ‘heads I win, tails the taxpayers lose’ — played an important role in the Asian crisis of 1997. The paragraphs below are taken from Corsetti, Pesenti and Roubini (1999), with minor changes.

“In interpreting the Asian meltdown, one should consider three different, yet strictly interrelated dimensions of the moral hazard problem at the corporate, financial, and international level. At the corporate level, political pressures to maintain high rates of economic growth had led to a long tradition of public guarantees to private projects, some of which were effectively undertaken under government control, directly subsidized, or supported by policies of directed credit to favored firms and/or industries. In light of the record of past government intervention, the production plans and strategies of the corporate sector largely overlooked costs and riskiness of the underlying investment projects. With financial and industrial policy enmeshed within a widespread business sector network of personal and political favoritism, and with governments that appeared willing to intervene in favor of troubled firms, markets operated under the impression that the return on investment was somewhat ‘insured’ against adverse shocks.

Such pressures and beliefs accompanied a sustained process of capital accumulation, resulting into persistent and sizable current account deficits. While common wisdom holds that borrowing from abroad to finance domestic investment should not raise concerns about external solvency – it could actually be the optimal course of action for under-capitalized economies with good investment opportunities – the evidence for the Asian countries in the mid-1990s highlights that the profitability of new investment projects was low.

Investment rates and capital inflows in Asia remained high even after the negative signals sent by the indicators of profitability. Consistent with the financial side of the moral hazard problem in Asia, the crucial factor underlying the sustained investment rates was excessive borrowing by national banks abroad, corresponding to high and excessive investment at home. Financial intermediation played a key role in channeling funds toward projects that were marginal if not outright unprofitable.

The adverse consequences of these distortions were crucially magnified by the rapid process of capital account liberalization and financial market deregulation in the region during the 1990s, which increased the supply-elasticity of funds from abroad. The extensive liberalization of capital markets was consistent with the policy goal of providing a large supply of low-cost funds to national financial institutions and the domestic corporate sector. The same goal motivated exchange rate policies aimed at reducing the volatility of the domestic currency in terms of the US dollar, thus lowering the risk premium on dollar-denominated debt.

The international dimension of the moral hazard problem hinged upon the behavior of international banks, which over the period leading to the crisis had lent large amounts of funds to the region's domestic intermediaries, with apparent neglect of the standards for sound risk assessment. Underlying such overlending syndrome may have been the presumption that short-term interbank cross-border liabilities would be effectively guaranteed by either a direct government intervention in favor of the financial debtors, or by an indirect bailout through IMF support programs. A very large fraction of foreign debt accumulation was in the form of bank-related short-term, unhedged, foreign-currency denominated liabilities: by the end of 1996, a share of short-term liabilities in total liabilities above 50% was the norm in the region. Moreover, the ratio of short-term external liabilities to foreign reserves – a widely used indicator of financial fragility – was above 100% in Korea, Indonesia and Thailand.”

**The banking crisis and Flood and Garber (1984)** What is the link between the banking crisis and the currency crisis? Corsetti, Pesenti and Roubini (1999) say:

“To satisfy solvency, the government must then undertake appropriate domestic fiscal reforms, possibly involving recourse to seigniorage revenues through money creation (that means, inflation, increase in  $P$ , that leads to the exchange rate devaluation). Speculation in the foreign exchange market, driven by expectations of inflationary financing, causes a collapse of the currency and anticipates the event of a financial crisis. Financial and currency crises thus become indissolubly interwoven in an emerging economy characterized

by weak cyclical performances, low foreign exchange reserves, and financial deficiencies eventually resulting into high shares of non-performing loans.”

In Flood and Garber (1984), the speculative attack results from expectations of inflationary financing. This pushes up the “shadow exchange rate” (in Flood and Garber, as PPP is assumed, that occurs instantaneously) and that leads to a speculative attack. Here, expectations of money creation come not through the expansion of domestic credit but from the fragility of the economy and the banking system (they point the expectations of bail-outs as key factor for that).

Other analysts will argue that those Asian countries had not bad economic fundamentals and that a self-fulfilling crisis in the form described by the second generation models happened. Radelet and Sachs (1998) is a well known example of a paper defending such view.

## 4 Sovereign debt and default

### 4.1 A model with exogenous interest rates

Before we go to models of sovereign default, it will be helpful to study a model with an exogenous relation between the interest rate and level of debt and no default.

**Model** Consider a small open economy with constant output  $y$ . Time is continuous and there is a measure-one continuum of agents. The economy can borrow or lend from abroad and  $b$  is the level of assets ( $-b$  is the level of debt). Consumption of the representative agent  $c$  is given by

$$\dot{b} = y + r(b)b - c \quad (20)$$

where  $r(\cdot)$  is a function that yields the interest rate. For  $b \geq 0$ ,  $r = r^*$ , we'll say  $r^* = 0$ . For  $b < 0$ , interest rates increase with the level of debt (so  $r'(b) < 0$ , since debt is  $-b$ ). The idea here is that a high level of debt makes default more likely. This will be formalized in the next section.

The representative agent maximizes.

$$U = \int_0^{\infty} e^{-\rho t} u(c_t) dt$$

where  $u(c)$  is the instantaneous utility of the representative agent. Let's suppose  $\rho > r^*$ , so that we get positive debt (negative assets) in equilibrium.

**Equilibrium** The current value Hamiltonian is

$$H = u(c_t) + \lambda_t (y + r(b)b - c)$$

The optimization conditions are

$$\frac{\partial H}{\partial c} = 0 \Rightarrow u'(c_t) = \lambda_t$$

and

$$\begin{aligned} \frac{\partial H}{\partial b} &= \rho \lambda_t - \dot{\lambda}_t \\ \Rightarrow \lambda (r'(b)b + r(b)) &= \rho \lambda_t - \dot{\lambda}_t \end{aligned}$$

The first condition implies

$$u''(c_t) \dot{c}_t = \dot{\lambda}_t$$

Hence

$$u'(c_t) (r'(b)b + r(b) - \rho) = -u''(c_t) \dot{c}_t$$

Thus

$$\frac{\dot{c}_t}{c_t} = \frac{r'(b)b + r(b) - \rho}{\theta} \quad (21)$$

where  $\theta$  is the relative risk aversion coefficient

$$\theta = \frac{-u''(c_t)c_t}{u'(c_t)}$$

Equations (20) and (21) characterize the equilibrium. In steady state,  $\dot{b} = 0$  and  $\dot{c} = 0$ . Hence

$$\begin{aligned} \rho &= r'(b)b + r(b) \\ c &= y + r(b)b \end{aligned} \quad (22)$$

The first equation shows that the steady state level of assets is negative,  $b < 0$ . Since  $\rho > r^*$ , we need  $r'(b)b > 0$  which only happens if  $b < 0$ . The idea is that the country borrows up to the point that the effect of an increase in the level of debt on the amount it has to pay to foreign borrowers ( $r'(b)b$ ) equates  $\rho - r(b)$ . The second equation shows that steady state consumption is given by output minus debt service.

Note that the expression in (22) can be written as:

$$\rho = \frac{\partial r}{\partial b} \frac{b}{r} r + r$$

which yields

$$\rho = r(1 + \epsilon) \implies r = \frac{\rho}{1 + \epsilon}$$

where

$$\epsilon = \frac{\partial r}{\partial b} \frac{b}{r}$$

is the elasticity of interest rates with respect to debt.

**An example** To get intuition, let's say

$$r(b) = -\frac{\gamma}{2}b$$

for  $b < 0$ , hence  $\gamma$  measures how interest rates are sensitive to the level of debt. Then  $r'(b) = -\gamma/2$ , and we get

$$b = -\frac{\rho}{\gamma}$$

the level of debt is smaller if interest rates are very sensitive to the level of debt. In equilibrium,  $r(b)$  is given by:

$$r = \frac{\rho}{2}$$

(which makes sense since the elasticity  $\epsilon = 1$ ). Thus

$$c = y - \frac{\rho^2}{\gamma^2}$$

which implies that steady state consumption is actually larger for high values of  $\gamma$ : if borrowing is too expensive, the domestic country borrows less, so steady state consumption is larger. In sovereign default models, it is essential to look at incentives to contract debt ex-ante, this already shows up here.

**The effect of unexpected shocks** The model can be used to analyse how the economy works when hit by unexpected shocks. That is not how we do research nowadays. The model has been solved assuming  $y$  and the function  $r(\cdot)$  are fixed, so technically a shock to parameters is a zero probability event in the model. So what is the point of studying 0-probability events? Well, even if agents attached a positive (and sizable) probability to shocks, any changes in  $y$  and  $r(\cdot)$  would still have an unexpected component and the effect of the unexpected component on the model is very similar to what we have here. Thus the intuition we get from this model carries on for more complicated models.

The dynamic system can be represented in a phase diagram (Figure 5). We see the loci  $\dot{b} = 0$  and  $\dot{c} = 0$ , the saddle path and the direction of the movement around the diagram.

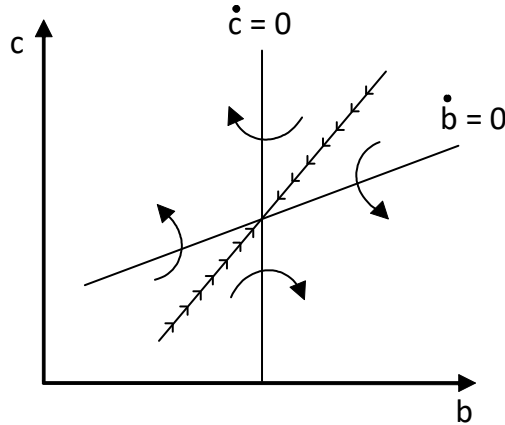


Figure 5: Phase diagram

Figure 6 shows the effect of a reduction in  $y$ : the locus  $\dot{b} = 0$  shifts down. Figure 7 shows the effect of an increase in world interest rates (say  $r$  goes up but its derivative is unchanged): the locus  $\dot{b} = 0$  shifts down and the locus  $\dot{c} = 0$  shifts to the right.

We can then analyze what happens when a temporary unexpected shock hits the economy.

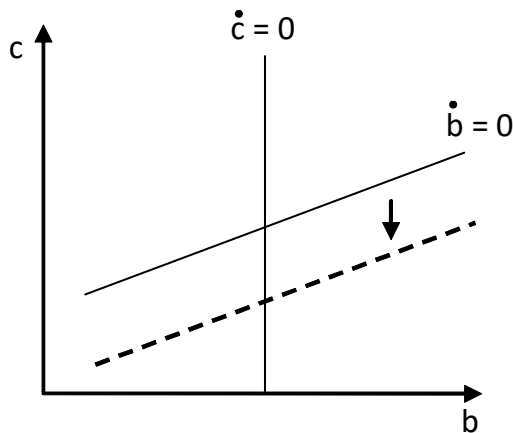


Figure 6: Effect of a reduction in  $y$

- Consider an unexpected and temporary negative shock to  $y$  that will last until time  $T$ : then the locus  $\dot{b} = 0$  shifts down. The shock is temporary so eventually the economy will get back to the saddle path depicted at Figure 5. Since  $b$  is a stock, it cannot jump, so  $c$  jumps down (but not all the way to the steady state equilibrium at Figure 6, it stays above that). Hence the economy moves to the left and up, and meet the original saddle path at time  $T$  (anticipated jumps to consumption do not happen, they are not optimal). Then the economy goes back to its steady state.

Intuitively, as the unexpected negative shock hits, consumption jumps down, but the economy starts to accumulate debt and consumption slowly comes back. When  $y$  goes back to its previous level, the economy repays the extra debt it had accumulated. More debt implies higher interest rates, which can be loosely interpreted here as higher default risk. Note that this is without making any assumption about how the level of output affects incentives for repaying.

- Consider an unexpected and temporary positive shock to  $r$  that will last until time  $T$  (say a constant is added to interest rates): the locus  $\dot{b} = 0$  shifts down and the locus  $\dot{c} = 0$  shifts to the right. The shock is temporary so eventually the economy will get back to the saddle path depicted at Figure 5. The effect is qualitatively similar to what we had before. The level of debt is a stock, it does not jump, but  $c$  jumps down and the economy starts moving up and to the left until it meets the original saddle path at time  $T$ . Then the economy goes back to its steady state.

The intuition is similar to what we have before, interest rate rates are high so the economy starts to consume less and goes back slowly to the previous level of con-



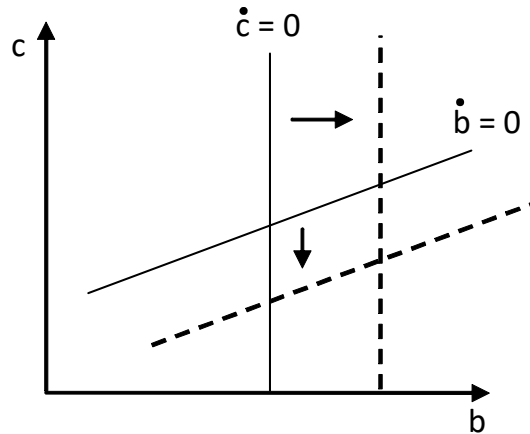


Figure 7: Effect of an increase in  $r(b)$

sumption. So it initially accumulates more debt and then pays it back while returns to its steady state.

## 4.2 Sovereign debt with no commitment to repay

The simple model above provides some intuition on the dynamics of an economy that wants to smooth consumption, can borrow from abroad but faces higher interest rates when the level of debt goes up. But how can we think about the relationship between economic variables (such as the level of debt), default risk and interest rates?

Eaton and Gersovitz (1981) propose a dynamic model of a small open economy that can borrow from abroad but cannot commit to repay debt that has become the standard framework for quantitative macro models of sovereign debt and default. We will now go through some of the key ideas in Eaton and Gersovitz (1981).

### 4.2.1 Endogenous default decision (Eaton and Gersovitz, 1981)

Sovereign debt is different from firms' or consumers' debt because the rules of the game are usually less clear. Thus it becomes important to understand incentives for a country to default or repay its sovereign debt.

The basic feature of models following Eaton and Gersovitz (1981) are:

- Time is discrete.
- Small open economy that can borrow from abroad but cannot commit to repay debt. In most cases, a planner maximizes utility of individuals.

- Default or debt payment is a choice. In the simplest cases, default means debt becomes zero.
- Markets are often incomplete, the sovereign country cannot issue debt contingent on the realization of a shock.
- There are foreign creditors, who are often risk-neutral investors, so they lend to the country and pin down the value of a bond,  $q$ :

$$q = \frac{1 - \pi}{1 + r^*}$$

where  $\pi$  is the probability of default (in this case default leads to zero debt) and  $r^*$  is the risk-free world interest rate. Naturally,  $\pi$  has to be endogenous.

We can thus write value functions:

$$\begin{aligned} V_p(d, X) &= \max_{d', W} \{u_p() + \beta E(V(d', X')|X)\} \\ V_d(X) &= \max_W \{u_d() + \beta E(V_d(d', X')|X)\} \\ V(d, X) &= \max \{V_p(d, X), V_d(X)\} \end{aligned}$$

where  $d$  is the level of debt,  $\beta$  is the time discount factor,  $X$  is a vector of state variables and  $W$  is a vector of control variables.  $V_p(d, X)$  is the value function in case the country chooses to repay. In that case, the domestic planner chooses debt next period ( $d'$ ) and whatever else ( $W$ ). The value function next period will be  $V(d', X')$  which is the maximum between  $V_p(d, X)$  and  $V_d(X)$ , the domestic country will then choose whatever is larger.  $V_d(X)$  is the value function in case of default, which in the simplest case is not a function of some level of debt because previous debt is wiped out in case of default. Note that here, once the country defaults, it is always in default state (next period value function is  $V_d(d', X')$ ). Most quantitative models assume an exogenous probability that the country regains access to international credit markets at every period. Note also that utility in case of default and debt payment ( $u_d()$  and  $u_p()$ ) are typically different, in most models default entails some punishment that affects something like output or trade.

The country is subject to a budget constraint. A country that chooses to pay debt has to pay  $d$  but can borrow  $qd'$ , both  $q$  and  $d'$  are endogenous.

One immediate implication of this model is that  $V_p(d, X)$  is decreasing in  $d$  (since the country has to pay debt  $d$ ) and  $V_d(X)$  is independent of  $d$  (since default implies all debt is wiped out). That implies there exists a level of debt  $\bar{d}(X)$ , which is typically a function of the state variables, such that it is optimal to repay debt if it is below that level. Figure 8 sums up this idea.

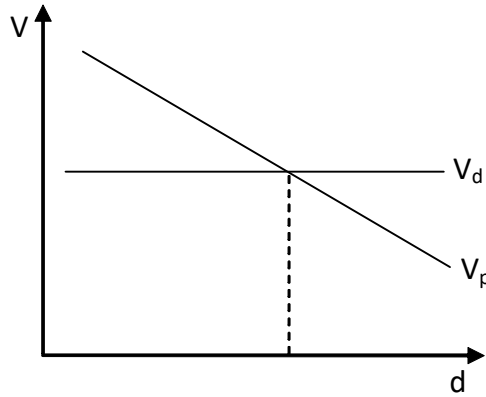


Figure 8: Choice of default

Shocks to the economy will affect the vector  $X$  and thus affect both  $V_p(d, X)$  and  $V_d(X)$ , so both curves will cross each other at different points implying a different maximum level of debt compatible with repayment.

#### 4.2.2 A deterministic example

For concreteness, let's analyze the simplest possible case, there is one good, the economy receives a constant endowment  $y$  in every period and world interest rates are constant and equal to  $r^*$ . If the country has ever defaulted, a fraction  $\gamma$  of the output is lost forever and the country loses access to international capital markets, so consumption is  $\gamma y$  forever (there is no investment or savings and no government spending). The time discount factor is  $\beta < (1 + r^*)^{-1}$ . Households get utility  $u(c)$  from consumption  $c$ .

Consumption in case of repayment is given by

$$c_p = y - d - qd'$$

and in case of default is given by

$$c_d = (1 - \gamma)y$$

hence value functions are

$$\begin{aligned} V_p(d) &= \max_{d'} \{u(c_p) + \beta V(d')\} \\ V_d &= u((1 - \gamma)y) + \beta V_d \\ V(d) &= \max \{V_p(d), V_d\} \end{aligned}$$

The only state variable is  $d'$ , which is known in advance. There is no uncertainty. Hence the value functions  $V_p(d')$  and  $V_d$  are known one period in advance, so it is known

whether the country will default or not.

Since there are no state variables, the level of debt  $\bar{d}$  that equates  $V_p(\bar{d})$  and  $V_d$  is constant. So the bond price schedule faced by the domestic economy is:

$$q = \begin{cases} \frac{1}{1+r^*} & \text{if } d' \leq \bar{d} \\ 0 & \text{if } d' > \bar{d} \end{cases}$$

since  $\pi$  is either 0 or 1. It makes no sense to issue bonds at price 0 (the domestic country gets nothing and then is punished because it defaulted).

What is  $\bar{d}$ ? Consider a country that always issues debt  $\bar{d}$ . Then

$$V_p(\bar{d}) = u\left(y - \bar{d} + \frac{1}{1+r^*}\bar{d}\right) + \beta V_p(\bar{d})$$

which yields

$$V_p(\bar{d}) = \frac{u\left(y - \frac{r^*}{1+r^*}\bar{d}\right)}{1-\beta}$$

The value function of a country that default on its debt implies

$$V_d = \frac{u((1-\gamma)y)}{1-\beta}$$

Equating  $V_p(\bar{d})$  and  $V_d$  and doing algebra leads to

$$\frac{\bar{d}}{y} = \frac{\gamma(1+r^*)}{r^*}$$

The maximum level of debt a country is willing to repay is proportional to its output because default punishment is proportional to output (by assumption); it is proportional to the default punishment; and it depends negatively on  $r^*$  since high interest rates imply that servicing debt is expensive, default becomes relatively more attractive.

### 4.3 Recent models

#### 4.3.1 Arellano (2008)

Arellano's quantitative dynamic stochastic small open economy model builds on Eaton and Gersovitz (RES 1981).

#### Setup of the model

- Small open economy

- Households are identical and have preferences given by

$$E_o \sum_{t=0}^{\infty} \beta^t u(c_t)$$

- and  $u(c_t)$  respects the Inada condition. In the numerical simulation:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

- Stochastic endowment:  $y$
- Government is benevolent and maximizes utility of households.
- One international asset: one period discount bonds ( $B$ ).
  - Price of bond:  $q$
  - Borrowing  $qB'$  implies repayment of  $B'$  next period
  - notation:  $B > 0$  means positive assets,  $B < 0$  means the country is indebted.
- Government chooses between defaulting or repaying
  - no commitment to repay
- If government chooses to repay, the resource constraint is:

$$c = y + B - qB'$$

- Default implies:
  - current debts are erased;
  - temporary exclusion from international financial markets (borrowing and lending);
  - output costs: endowment is lower, equal to  $y^{def} = h(y) \leq y$ ,  $h$  is an increasing function;
  - so:

$$c = y^{def}$$

- Country re-enters the financial market with an exogenous constant probability  $\theta$ .
- Probability of default:  $\delta$
- Creditors are risk-neutral and behave competitively.

- International risk-free interest rate:  $r$
- An arbitrage condition pins down  $q$ :

$$\begin{aligned} R^f &= R^d \\ 1 + r &= \frac{1}{q}(1 - \delta) \\ \Rightarrow q &= \frac{1 - \delta}{1 + r} \end{aligned}$$

- Timing:

- government starts with assets  $B$ , observes realization of endowment  $y$  and decides about repaying or not;
- if the government decides to repay, borrowing takes place.
- consumers consume.

The value function depends on whether the country repays or defaults on its debt. Denote by  $v^c$  the value function conditional on repayment and by  $v^d$  the value function conditional on default.

The government chooses what is best for the agents. Suppose we start the period at state  $(B, y)$ . Denote the value function by  $v^o(B, y)$ . Then:

$$v^o(B, y) = \max \{v^c(B, y), v^d(y)\}$$

If the country decides to repay, than the value function is given by:

$$v^c(B, y) = \max_{B'} \left\{ u(c) + \beta \int_{y'} v^o(B', y') f(y'|y) dy' \right\}$$

where  $c = y + B - qB'$ .

If the country chooses to default, the value function is given by:

$$v^d(y) = u(y^{def}) + \beta \int_{y'} [\theta v^o(0, y') + (1 - \theta)v^d(y')] f(y'|y) dy'$$

**Equilibrium** In a recursive equilibrium:

1. Risk-neutral creditors are indifferent between lending to the domestic country or not (so  $q$  is obtained through the above arbitrage condition).
2. Government maximizes.
3. Households eat.

All the action comes from the government's decision.

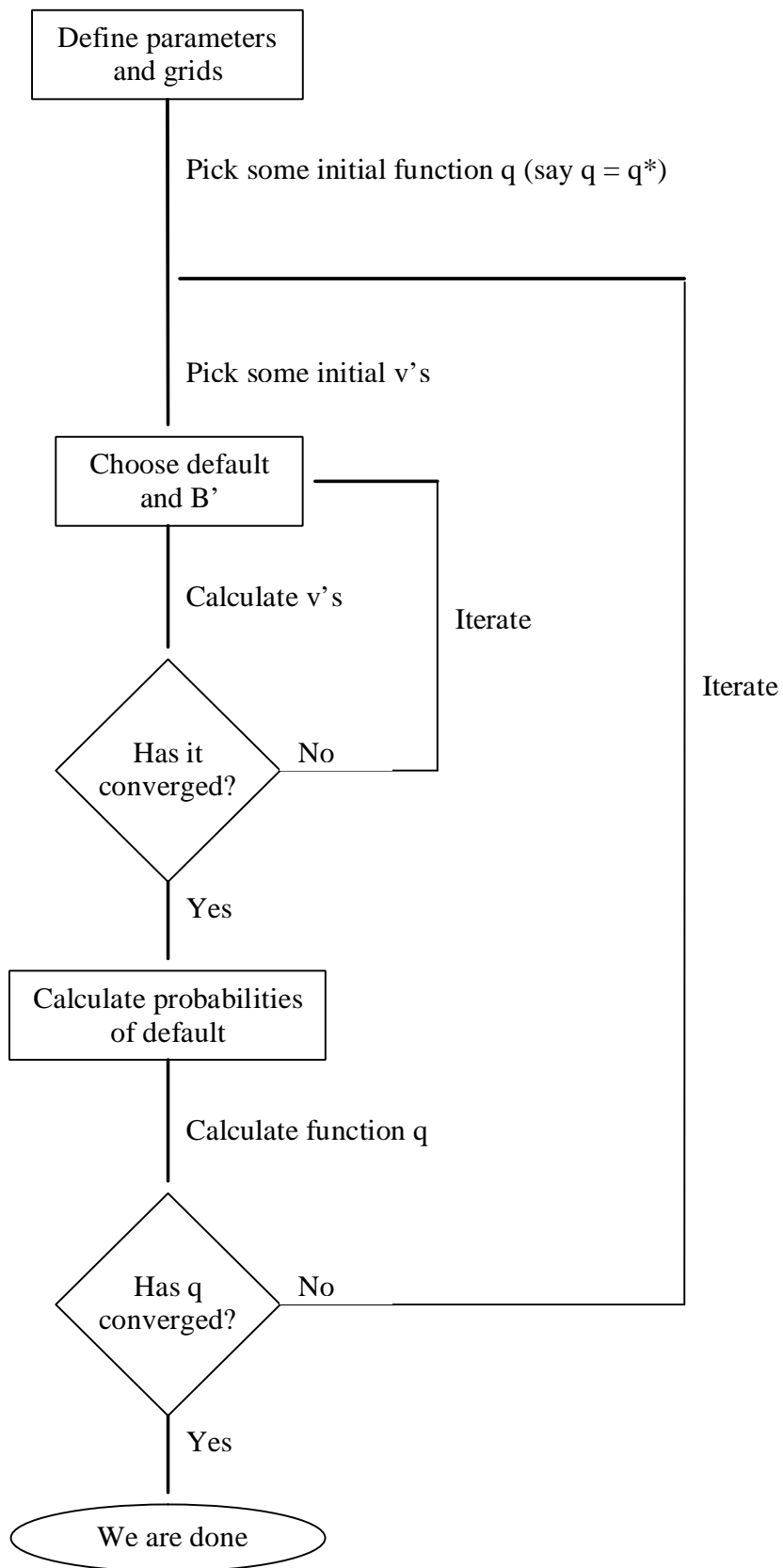


Figure 9: Flow diagram for Arellano (2008)

**Computation of equilibrium** The problem here is partial equilibrium in the sense that we do not model the outside world (foreign interest rates are taken as given). However, prices depend on the probabilities of default (arbitrage condition for the risk-neutral creditors) and although the government internalizes the effects of its actions on the price of debt, we can't solve for it directly. We have to find  $q$  as a fixed point —  $q$  is a function.

Figure 9 shows the basic structure of an algorithm to compute the equilibrium of the model.

**Results** Negative shocks to  $y$  might prompt default or might increase default risk. So default occurs in bad times, interest rates increase in recessions (owing to the default premium). See the figures in the paper.

#### 4.3.2 Cuadra, Sanchez and Sapriza (2010)

The point this paper wants to explain is the following: emerging market economies typically exhibit a procyclical fiscal policy, expenditures fall and taxes rise in recessions. Moreover, they experience countercyclical default risk. Cuadra, Sanchez and Sapriza (2010) develop a dynamic model with endogenous fiscal policy and sovereign default to account for that. Here I present a simplified version of that model.

**The model** A small open economy can borrow from abroad but cannot commit to repay debt. A representative agent gets endowment  $y_t$ , with probability density function  $f$  and support  $[y_L, y_H]$ . The representative agent maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) + v(g_t))$$

where  $\beta < 1$ . Private consumption ( $c_t$ ) and government spending ( $g_t$ ) are given by:

$$\begin{aligned} c_t &= (1 - \tau_t)y_t \\ g_t &= \tau_t y_t + q_t d_{t+1} - d_t \end{aligned}$$

where  $\tau_t$  is the tax rate,  $d_t$  is debt maturing at  $t$  (contracted at  $t - 1$ ) and  $q_t$  is the endogenous price of debt. Output  $Y_t$  is given by:

$$Y_t = \begin{cases} y_t & \text{if default has never happened} \\ g(y_t) & \text{otherwise} \end{cases}$$

where  $g(\cdot)$  is such that default happens in bad times (there is less punishment for default when  $Y_t$  is low). The government chooses  $\tau_t$  and  $d_{t+1}$ , where

$$q_t = \frac{\pi_t}{1 + r^*}$$



and  $\pi_t$  is the probability of repayment in the following period.

The value function if the country repays debt is:

$$V_p(d, y) = \max_{\tau, d'} \{u((1 - \tau)y) + v(\tau y + qd' - d) + \beta EV(d', y'|y)\}$$

and if the country defaults, it is

$$V_d(y) = \max_{\tau} \{u((1 - \tau)g(y)) + v(\tau g(y)) + \beta EV_d(y'|y)\}$$

and

$$V(d, y) = \max \{V_p(d, y), V_d(y)\}$$

Now let's take for granted that default occurs if and only if  $y_t < \bar{Y}(d_t)$ , where  $\bar{Y}(d)$  is the value of  $Y_t$  that makes the government indifferent between defaulting and paying the debt,  $\bar{Y}(d)$  is increasing in  $d$  (in the paper that occurs in equilibrium owing to the assumptions on the function  $g$ ). Since at  $\bar{Y}(d)$  the country is indifferent between repaying and defaulting,

$$V_p(d, \bar{Y}(d)) = V_d(\bar{Y}(d))$$

Since default occurs next period if and only if  $Y_t < \bar{Y}(d')$ ,

$$EV(d', y'|y) = \int_{y_L}^{\bar{Y}(d')} V_d(y) f(y'|y) dy' + \int_{\bar{Y}(d')}^{y_H} V_p(d', y) f(y'|y) dy'$$

**First order conditions** Consider a country that has not defaulted (so risk of default is an issue). The first order condition with respect to  $\tau$  implies

$$\frac{\partial u}{\partial c} = \frac{\partial v}{\partial g}$$

and the first order condition with respect to  $d'$  yields

$$\frac{\partial v}{\partial g} \left( \frac{\partial(qd')}{\partial d'} \right) + \beta \int_{\bar{Y}(d')}^{y_H} \frac{\partial V_p(d', y)}{\partial d'} f(y'|y) dy' + \beta \frac{\partial \bar{Y}(d')}{\partial d'} (V_d(\bar{Y}(d')) - V_p(d', \bar{Y}(d'))) f(\bar{Y}(d')|y) = 0$$

Since  $V_d(\bar{Y}(d')) = V_p(d', \bar{Y}(d'))$  (because the country is indifferent between paying and defaulting in state  $(d', \bar{Y}(d'))$ ), that becomes

$$\frac{\partial v}{\partial g} q(1 - \varepsilon) + \beta \int_{\bar{Y}(d')}^{y_H} \frac{\partial V_p(d', y)}{\partial d'} f(y'|y) dy' = 0$$

where  $\varepsilon$  is the absolute value of the elasticity of the price of debt with respect to  $d'$ ,

$$\varepsilon = -\frac{\partial q}{\partial d'} \frac{d'}{q}$$

and since

$$\frac{\partial V_p(d, y)}{\partial d} = -\frac{\partial v}{\partial g}$$

we get

$$\frac{\partial v}{\partial g} \frac{\pi}{1 + r^*} (1 - \varepsilon) = \beta \int_{\bar{Y}(d')}^{y_H} \frac{\partial u}{\partial g} \Big|_{g=\tau'y'-d'+q'd''} f(y'|y) dy'$$

where the integral term is the expected value of the marginal utility of government spending conditional on repayment times the probability of repayment. I will write that as

$$\int_{\bar{Y}(d')}^{y_H} \frac{\partial v}{\partial g} \Big|_{g=\tau'y'-d'+q'd''} f(y'|y) dy' = \pi E_{y'} \left( \frac{\partial v}{\partial g} \Big|_{y' \geq \bar{Y}(d')} \right)$$

so

$$\frac{\partial v}{\partial g} (1 - \varepsilon) = (1 + r^*) \beta E_{y'} \left( \frac{\partial v}{\partial g} \Big|_{y' \geq \bar{Y}(d')} \right)$$

Note that the probability of default per se has no first order impact in the sense that a lower  $\pi$  makes debt more expensive but debt is also less likely to be repayed so the  $\pi$  at both sides of the equation cancels out. But  $\varepsilon$  has an important effect.

Suppose the case where default is not possible, which means  $\pi = 1$  and  $\varepsilon = 0$ . Then

$$\frac{\partial u}{\partial g} = (1 + r^*) \beta E_{y'} \left( \frac{\partial v}{\partial g} \right)$$

which is a pretty standard result. But now suppose  $\varepsilon$  is large ( $1 - \varepsilon$  is small). Then the government wants a large present marginal utility of government spending (and so a large marginal utility of consumption as well). That means small levels of consumption and government spending and, consequently, little borrowing. The reason is that borrowing one extra unit decreases the price of debt (increases the interest rate paid by the government), so it is better to decrease domestic expenditure and borrow little.

At times when default risk is more important, the negative effect of the level of debt on the price of debt kicks in, then it is optimal for the government to choose small  $g$  and high  $\tau$  (low consumption). Procyclical fiscal policy is thus a consequence of counter-cyclical default risk.

### 4.3.3 Which shocks matter?

Guimaraes (2011) analyses whether sovereign default episodes can be seen as contingencies of optimal international lending contracts. The model also considers a small open economy without commitment to repay debt. The point is that shocks to world interest rates have strong effects on the incentives for default – much stronger than output shocks if the cost of default in terms of output loss is proportional to the level of output. Here I present a simplified version of the argument.

**The model** An open economy can borrow from abroad, but cannot commit to repay its debts. The economy is populated by a continuum of infinitely lived agents whose preferences are aggregated to form the usual representative agent utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

Default implies a permanent output cost,  $\gamma$ . The fraction of output lost due to default is  $\gamma$ , so production is given by:

$$y_t = \begin{cases} y_t, & \text{if no default} \\ (1 - \gamma).y_t, & \text{if default} \end{cases}$$

Default also implies permanent exclusion from international capital markets. Default costs are permanent, which captures the loss that a country suffers by taking an antagonistic position towards the rest of the world and never repaying its debts. In the model this is out-of-equilibrium behaviour, which corresponds to never observing such action in reality.

There is a continuum of risk-neutral lenders that, in equilibrium, lend to the country as long as the expected return on their assets is not lower than the risk-free interest rate in international markets,  $r^*$ . The price of a bond that delivers one unit of the good next period with certainty,  $(1 + r^*)^{-1}$ , will be denoted  $q^*$ . There is a maximum amount of debt the country can contract that prevents it from running Ponzi schemes but it is never reached in equilibrium.

The economy's flow budget constraint is then given by:

$$c_t = \begin{cases} y_t - d_t + q_t d_{t+1}, & \text{if it has never defaulted} \\ (1 - \gamma)y_t, & \text{if it has ever defaulted} \end{cases}$$

There are two states,  $s \in \{h, l\}$ . We will explicitly condition debt repayment on the state.

**Stochastic interest rates** Let's analyze the case of stochastic  $q^*$ . The price of a riskless bond in international markets is  $q^{*h}$  in the high state and  $q^{*l}$  in the low state,  $q^{*h} > q^{*l}$ . The economy switch to the other state with probability  $\psi$ , so the probability  $s_t = s_{t-1}$  is  $1 - \psi$ ; with probability  $\psi$ ,  $s_t \neq s_{t-1}$ .

We want to understand what is the maximum incentive compatible level of debt at each state? If the country borrows up to the maximum it can, and lenders can always receive the maximum payment the country is willing to pay, how much debt falls when the economy goes from the high to the low state?

A risk-neutral creditor that lends  $q^*d'$  must get an expected repayment equal to  $d'$ . Denote by  $d^h$  and  $d^l$  the repayment conditional on high and low state, respectively, and  $\Delta d = d^h - d^l$ . If  $s_{t-1} = h$ , a country that borrowed  $q^{*h}d'$  has debt  $d^h$  if  $s_t = h$  and  $d^l$  if  $s_t = l$  such that  $d^h(1 - \psi) + d^l\psi = d'$ .<sup>2</sup> If  $s_{t-1} = l$ , a country borrowing  $q^{*l}d'$  has debt  $d^h$  if  $s_t = h$  and  $d^l$  if  $s_t = l$  such that  $d^l(1 - \psi) + d^h\psi = d'$ .<sup>3</sup>

In each period, the central planner chooses between repaying or defaulting. Each option yields a different value function and the planner chooses the maximum of the two:

$$V(q^*, d) = \max \{V_{pay}(q^*, d), V_{def}\}$$

The value functions conditional on repayment are:

$$\begin{aligned} V_{pay}^h(d) &= \max_{d^h} \{u(y - d + q^{*h} [(1 - \psi)d^h + \psi d^l]) + \beta [(1 - \psi)V_{pay}^h(d^h) + \psi V_{pay}^l(d^l)]\} \\ V_{pay}^l(d) &= \max_{d^h} \{u(y - d + q^{*l} [(1 - \psi)d^l + \psi d^h]) + \beta [(1 - \psi)V_{pay}^l(d^l) + \psi V_{pay}^h(d^h)]\} \end{aligned}$$

where  $d^h$  and  $d^l$  are the levels of debt in the high and low state, respectively.

In case of default, the value function in both states is:

$$V_{def} = u((1 - \gamma)y) + \beta V_{def} = \frac{u((1 - \gamma)y)}{1 - \beta}$$

It makes no difference whether foreign interest rates are low or high if the country is excluded from international financial markets.

**Equilibrium** The incentive compatible level of debt at the high state  $d^h$  is given by  $V_{pay}^h(d^h) = V_{def}$ , and the incentive compatible level of debt at the low state  $d^l$  is given by  $V_{pay}^l(d^l) = V_{def}$ . Hence,  $V_{pay}^h(d^h) = V_{pay}^l(d^l)$ . Thus

$$u(y - d^h + q^{*h} [(1 - \psi)d^h + \psi d^l]) = u(y - d^l + q^{*l} [(1 - \psi)d^l + \psi d^h])$$

and doing algebra we get to

$$\frac{d^h - d^l}{\bar{d}} = \frac{q^{*h} - q^{*l}}{1 - \bar{q}(1 - 2\psi)} \quad (23)$$

where  $\bar{d} = (d^h + d^l) / 2$  and  $\bar{q} = (q^h + q^l) / 2$ .

**Stochastic technology** Now fix the world interest rates and allow  $y$  to fluctuate between  $y^h$  in the high state and  $y^l$  in the low state,  $y^h > y^l$ . The value functions conditional on

<sup>2</sup> Hence,  $d^h = d' + \psi\Delta d$  and  $d^l = d' - (1 - \psi)\Delta d$ .

<sup>3</sup> Hence,  $d^l = d' - \psi\Delta d$  and  $d^h = d' + (1 - \psi)\Delta d$ .

repayment are:

$$\begin{aligned} V_{pay}^h(d^h) &= \max_{d^h} \{u(y^h - d + q^* [(1 - \psi)d^h + \psi d^l]) + \beta [(1 - \psi)V^h(d^h) + \psi V^l(d^l)]\} \\ V_{pay}^l(d^l) &= \max_{d^l} \{u(y^l - d + q^* [(1 - \psi)d^l + \psi d^h]) + \beta [(1 - \psi)V^l(d^l) + \psi V^h(d^h)]\} \end{aligned}$$

Should the country go into default, the value functions are:

$$\begin{aligned} V_{def}^h &= u((1 - \gamma)y^h) + \beta [(1 - \psi)V_{def}^h + \psi V_{def}^l] \\ V_{def}^l &= u((1 - \gamma)y^l) + \beta [(1 - \psi)V_{def}^l + \psi V_{def}^h] \end{aligned}$$

Making  $V_{pay}^h(d^h) = V_{def}^h$ , we get:

$$d^h - q^* [\psi d^l + (1 - \psi)d^h] = \gamma y^h \quad (24)$$

Analogously  $V_{pay}^l(d^l) = V_{def}^l$  yields:

$$d^l - q^* [\psi d^h + (1 - \psi)d^l] = \gamma y^l \quad (25)$$

Subtracting (25) from (24), we get:

$$(d^h - d^l) [1 - q^*(1 - 2\psi)] = \gamma(y^h - y^l) \quad (26)$$

Summing (25) and (24) and manipulating, we get:

$$\gamma = \frac{\bar{d}}{\bar{y}}(1 - q^*)$$

Using this value of  $\gamma$  on the above equation yields:

$$\frac{d^h - d^l}{\bar{d}} = \frac{(1 - q^*)}{1 - q^*(1 - 2\psi)} \frac{y^h - y^l}{\bar{y}} \quad (27)$$

**Stochastic interest rates × stochastic technology** Compare the expressions for  $(d^h - d^l) / \bar{d}$  in (23) and (27): debt relief generated by reasonable fluctuations in productivity is at least an order of magnitude below that generated by shocks to world interest rates.

#### 4.3.4 IMF and fiscal adjustment

Gonçalves and Guimaraes (2015) study time consistency of fiscal policy and the role of the IMF in a model of sovereign default. I present here a simplified version of that model.

**The model** An open economy can borrow from abroad, but cannot commit to repay its debts. The economy is populated by a continuum of infinitely lived agents whose preferences are aggregated to form the usual representative agent utility function:

$$\sum_{t=0}^{\infty} \beta^t (u(c_t) + g_t)$$

subject to  $g_t \geq 0$  (and  $c_t \geq 0$ ).

In each period, the country's representative agent receives a stochastic endowment  $y$  drawn from a probability density function  $f$ , which is continuous with full support in the  $[y_L, y_H]$  interval, with  $y_H > y_L > 0$  and iid. It is also assumed that  $u'(0) \rightarrow \infty$  and  $u'(y_L) < 1$ .

Default implies a permanent output cost,  $\gamma$ . The fraction of output lost due to default is  $\gamma$ , so production is given by:

$$y_t = \begin{cases} y_t, & \text{if no default} \\ (1 - \gamma).y_t, & \text{if default} \end{cases}$$

Default also implies permanent exclusion from international capital markets.

There is a continuum of risk-neutral lenders that, in equilibrium, lend to the country as long as the expected return on their assets is not lower than the risk-free interest rate in international markets,  $r^*$ . The price of a bond that delivers one unit of the good next period with certainty,  $(1 + r^*)^{-1}$ , will be denoted  $q^*$ .

The sequence of events in a given discrete time period is the following:

1.  $y$  is revealed;
2. decisions about defaulting on maturing debt  $d$  and floating new debt obligations  $d'$  are made;
3. the tax rate prevailing next period  $\tau'$  is chosen.

Taxes are chosen one period in advance. The private sector has no access to external capital markets and hence private consumption equals net income:

$$c = \hat{y}(1 - \tau)$$

Government spending is given by:

$$\begin{aligned} g &= \tau \hat{y} - d + qd' && \text{if the debtor has access to capital markets} \\ g &= \tau \hat{y} && \text{otherwise} \end{aligned}$$

**Value functions** Debt and default decisions are made after  $y$  is observed but before  $\tau'$  has been chosen, reflecting the idea that the country cannot commit to a certain level of taxes when it issues debt.

The value function associated with repaying debt is:

$$V_p(\tau, d, y) = \max_{d'} \{u([1 - \tau]y) + \tau y - d + qd' + \beta EV(\tau', d', y')\} \quad (28)$$

where  $\tau'$  maximizes

$$u([1 - \tau]y) + \tau y - d + qd' + \beta EV(\tau', d', y') \quad (29)$$

taking  $d'$  as given.  $V$  is the maximum of two value functions:

$$V(\tau', d', y') = \max \{V_p(\tau', d', y'), V_d(\tau', y')\}$$

If the government opts to default, the value function is:

$$V_d(\tau, y) = \max_{\tau'} \{u([1 - \tau]y) + \tau y + \beta EV_0(\tau', y')\} \quad (30)$$

where

$$V_0(\tau', y') = \max_{\tau''} \{u([1 - \tau']\hat{y}') + \tau'\hat{y}' + \beta EV_0(\tau'', y'')\}$$

and  $\hat{y}' = (1 - \gamma)y'$ . The default punishment kicks in one period after the default decision. As a tie-breaking convention, we will suppose that the country repays debt if indifferent.

**Default when tax revenues are low** When the constraint  $g \geq 0$  is slack, the derivatives of the value functions with respect to  $y$  are

$$\frac{dV_p}{dy} = \frac{dV_d}{dy} = u'(y(1 - \tau))(1 - \tau) + \tau$$

Hence when the constraint  $g \geq 0$  is not binding, there is never default.

The constraint  $g \geq 0$  can be written as  $\tau y - d + qd' \geq 0$ . Since  $\tau$ ,  $y$  and  $d$  are predetermined, only  $qd'$  can affect the constraint. Differentiating  $qd'$  with respect to  $d'$  yields

$$\frac{\partial(qd')}{\partial d'} = q(1 - \epsilon) \quad \text{where} \quad \epsilon = -\frac{\partial q}{\partial d'} \frac{d'}{q}$$

The probability of repayment is (weakly) decreasing in the level of debt. Hence extra debt  $\Delta d'$  will tend to increase the inflow of resources by less than  $q\Delta d'$ .

Default only occurs when the constraint  $g \geq 0$  is binding. That happens when tax revenues are low. In particular,

1. Default can only happen in bad times: for given  $\tau$  and  $d$ , if there is default for some  $y \in [y_L, y_H]$  then there is default if and only if  $y < \tilde{y}$  for some  $\tilde{y} \in [y_L, y_H]$ .

2. Default is more likely when  $\tau$  is smaller: for given  $y$  and  $d$ , if there is default for some  $\tau \in [0, 1]$  then there is default if and only if  $\tau < \tilde{\tau}$  for some  $\tilde{\tau} \in [0, 1]$ .
3. If the probability of default next period is positive, than it is strictly decreasing in  $\tau'$ .

The proof for the first 2 statements is basically the same. The key point is that when the constraint  $g \geq 0$  is locally binding,

$$\frac{dV_p(\tau, d, y)}{dy} > u'(y(1 - \tau))(1 - \tau) + \tau \quad (31)$$

I will show that. Since the constraint on  $g$  is binding,

$$V_p(\tau, d, y) = \max_{d'} \{u(y(1 - \tau)) + \beta E[V(\tau', d', y')]\}, \text{ with } qd' = \phi d - \tau y$$

thus

$$\frac{\partial V_p(\tau, d, y)}{\partial y} = u'(y(1 - \tau))(1 - \tau) + \beta \frac{\partial E[V(\tau', d', y')]}{\partial d'} \frac{\partial d'}{\partial y} \Big|_{d'=(\phi d - \tau y)/q} \quad (32)$$

We need show that the second term is larger than  $\tau$ . Note that

$$\frac{\partial(qd')}{\partial y} \Big|_{d'=(\phi d - \tau y)/q} = \frac{\partial(qd')}{\partial d'} \frac{\partial d'}{\partial y} \Big|_{d'=(\phi d - \tau y)/q}$$

and since

$$\frac{\partial(qd')}{\partial y} \Big|_{d'=(\phi d - \tau y)/q} = -\tau \quad \text{and} \quad \frac{\partial(qd')}{\partial d'} = q(1 - \epsilon)$$

we have

$$\frac{\partial d'}{\partial y} \Big|_{d'=(\phi d - \tau y)/q} = -\frac{\tau}{q(1 - \epsilon)} \quad (33)$$

Moreover, since the constraint is locally binding,  $d' = (\phi d - \tau y) / q$  and the derivative of the value function with respect to  $d'$  is negative (the country would prefer a smaller  $d'$  leading to a smaller  $g$  but has to borrow enough to get  $g = 0$ ),

$$\begin{aligned} \frac{\partial(qd')}{\partial d'} + \beta \frac{\partial E[V(\tau', d', y')]}{\partial d'} &< 0 \\ q(1 - \epsilon) &< -\beta \frac{\partial E[V(\tau', d', y')]}{\partial d'} \end{aligned}$$

Combining this with (33) yields:

$$\beta \frac{\partial E[V(\tau', d', y')]}{\partial d'} \frac{\partial d'}{\partial y} \Big|_{d'=(\phi d - \tau y)/q} > \tau$$

Using (32), that implies the condition in (31).



The reasoning leading to the second statement is essentially the same.

The intuition is the following: tax revenues are given by  $\tau y$ . When they are low, the government constraint binds with  $g = 0$ , which leads to a larger reduction in  $V_p$  in comparison to  $V_d$ . The optimal unconstrained borrowing  $d'$  is not feasible. The constrained level of borrowing is suboptimally large in case of repayment. Higher financing needs translate into a larger probability of default, which drives down the price of debt.

That does not imply that the debtor will be transferring more money to creditors, because creditors always break-even in expected terms. From an accounting perspective, the increase in interest rates and the decrease in the probability of repayment cancel each other. However, from an economic perspective, the decrease in the probability of repaying increases the probability of facing output costs in the future. Hence an increase in financing needs effectively renders the option of repaying relatively more costly.

**The time inconsistency problem** Focus on the case the country has access to capital markets. The optimal tax rate  $\tau'$  in equilibrium comes from:

$$\frac{\partial EV(\tau', d', y')}{\partial \tau'} = 0 \quad (34)$$

Now suppose the government can commit to some tax rate equal to  $\hat{\tau}$  when it issues debt  $d'$  at the current period.<sup>4</sup> The value function becomes

$$V_p^C(\tau, d, y) = \max_{\hat{\tau}, d'} \{u([1 - \tau]y) + \tau y - d + qd' + \beta EV(\hat{\tau}, d', y')\} \quad (35)$$

where  $q$  is now a function of both  $d'$  and  $\hat{\tau}$ .

One implication of the model is that if the probability of default next period is positive and the debtor can commit to a tax rate  $\hat{\tau}$  when issuing debt  $d' > 0$ ,

$$\frac{\partial q}{\partial \hat{\tau}} > 0$$

An increase in  $\hat{\tau}$  increases the probability of repayment and that is reflected in the price of debt  $q$ . Higher taxation means less refinancing needs and thus smaller chances of defaulting in the future.

We now show that if the debtor could commit to a certain tax rate  $\hat{\tau}$ , the chosen  $\hat{\tau}$  would be larger than  $\tau'$  given by (34). Now,  $\hat{\tau}$  maximizes the expression for  $V_p^C(\tau, d, y)$  in (35), taking into account the effect of  $\hat{\tau}$  on  $q$ .

The solution to the maximization problem for  $\hat{\tau}$  depends on whether the constraint  $g \geq 0$  binds. Suppose first it does not bind. Then  $\tau y - d + qd' > 0$  and the first order

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<sup>4</sup>For expositional simplicity, I assume there will be no opportunity for commitment in the future.

condition with respect to  $\hat{\tau}$  yields:

$$d' \frac{\partial q}{\partial \hat{\tau}} + \beta \frac{\partial EV(\hat{\tau}, d', y')}{\partial \hat{\tau}} = 0$$

The choice of  $\tau'$  with no commitment is given by (34). When the government can commit to a tax rate  $\hat{\tau}$  before issuing debt,  $\hat{\tau}$  has an extra positive effect on the value function, since it positively affects the debt price  $q$ .

Now suppose the constraint  $g \geq 0$  binds so that  $\tau y - d + qd' = 0$ . Then  $d' = (\phi d - \tau y) / q$  and

$$\frac{\partial d'}{\partial \hat{\tau}} = -\frac{(\phi d - \tau y)}{q^2} \frac{\partial q}{\partial \hat{\tau}}$$

hence the first order condition with respect to  $\hat{\tau}$  implies

$$\begin{aligned} \beta \left( \frac{\partial EV(\hat{\tau}, d', y')}{\partial d'} \frac{\partial d'}{\partial \hat{\tau}} + \frac{\partial EV(\hat{\tau}, d', y')}{\partial \hat{\tau}} \right) &= 0 \\ \beta \left( -\frac{(\phi d - \tau y)}{q^2} \frac{\partial q}{\partial \hat{\tau}} \int_{\bar{y}}^{y_H} \frac{\partial V_p(\hat{\tau}, d', y')}{\partial d'} f(y') dy' + \frac{\partial EV(\hat{\tau}, d', y')}{\partial \hat{\tau}} \right) &= 0 \end{aligned}$$

In both cases, we are adding a positive term to the marginal effect of increasing  $\hat{\tau}$ . If commitment is feasible, a pledge to implement a certain tax rate  $\hat{\tau}$  will be fully incorporated in bond prices, which increases the incentives for additional fiscal adjustment. The debtor benefits from higher taxes since these mean less frequent defaults.

**A commitment device** A contingent debt contract that entitles the debtor to receive a transfer  $T$  contingent on choosing  $\tau^* + \Delta\tau$  improves welfare in the debtor country and leaves the lenders unaffected. We can see that as a loan of  $T/(1+r^*)$  from the borrower to the lender at the risk-free rate, with a condition that the borrower will only be repaid if it chooses a tighter fiscal stance.

That has a positive effect: it works as a commitment device for the debtor. It also has a negative effect: since the debtor receives  $T$  only in the following period, it might have to borrow more in the current period to roll over its debt (in case the  $g \geq 0$  constraint is binding), and borrowing is expensive. However, for a small  $\Delta\tau$ , the transfer  $T$  is of second order of magnitude because the debtor is close to indifference between  $\tau^*$  and  $\tau^* + \Delta\tau$ , so the negative effect is dominated by the positive effect.

A private contract along these lines requires that lenders commit to a transfer contingent on a certain level of fiscal adjustment, which has to be chosen and then verified. That requires some degree of commitment and a large amount of effort. Individual creditors would have incentives to free ride on others' monitoring efforts. Overcoming the time inconsistency problem discussed above through private contracting seems to be unfeasible.

However, the problem can be solved by a big long-term player that is able to (i) choose and verify the optimal level of fiscal adjustment; (ii) commit to transfer resources to the debtor after observing the implementation of the chosen level of fiscal policy; and (iii) take resources from the lenders to fund this transfer.

## 5 Debt crises and coordination

The models on sovereign default along the lines of Eaton and Gersovitz (1981) are all about solvency and incentives for repayment. However, sometimes countries might go through liquidity crises: they want to roll over their debts and will be able and willing to pay the debt later, but cannot roll it over. We start with a simple model of liquidity crises that illustrates the problem. Then we discuss models where coordination plays a key role.

### 5.1 Liquidity crises

There is a measure-one continuum of foreign investors, each one has invested 1 dollar in a small open economy at time  $t = 0$ . The deal is the following: they can withdraw their money at  $t = 1$  and get their dollar back or can wait until  $t = 2$ . The resources are invested in a technology that yields  $R$  for every unit invested.

However, withdrawing their investments at  $t = 1$  reduces the return to investment in this economy. Let  $x$  be the fraction of investors that withdraw their money at  $t = 1$ . The gross return in the economy at  $t = 2$  is given by:

$$(1 - x)R + x\lambda R$$

where  $\lambda < 1$ . So if noone withdraws at  $t = 1$  ( $x = 0$ ), the return is  $R$  indeed, but if everyone withdraws early ( $x = 1$ ), early liquidation of projects reduces returns to  $\lambda R$  only.

The amount of resources available at  $t = 2$  is given by

$$\max\{0, (1 - x)R + x\lambda R - x\}$$

which is divided among the remaining  $1 - x$  agents. For simplicity, the time discount factor is 1.

**Equilibrium** Suppose  $x = 0$ . Then each agent receives  $R$  in the second period. Thus it is optimal to withdraw early only if  $R \leq 1$ .

However, suppose  $x = 1$ . An individual waiting until  $t = 2$  will receive 0 if  $\lambda R < 1$ . Hence it is optimal to withdraw early if  $R < 1/\lambda$ .

Consequently:

- For  $R < 1$ ,  $x = 1$  in the unique equilibrium.
- For  $R > 1/\lambda$ ,  $x = 0$  in the unique equilibrium.

- For  $R \in [1, 1/\lambda]$ , there are multiple equilibria. In one equilibrium,  $x = 1$ , in the other equilibrium,  $x = 0$ .

### 5.1.1 IMF intervention

Now suppose the IMF can lend any amount to this economy.

Suppose the IMF lends  $x$  to this economy at  $t = 1$ . Then there are no liquidation costs. At  $t = 2$ , the economy gets  $R$  and needs to pay those who had not withdrawn early ( $1 - x$  agents) plus the IMF (that has lent  $x$ ). Suppose the IMF gets the same as each creditor per dollar it lent. Then each creditor gets  $R$  and the IMF gets  $Rx$ .

If  $R < 1$ , the IMF loses money, so it will not lend. But for  $R > 1$ , the IMF lends to the economy and avoids a liquidity crisis. Anticipating that, private agents have no incentives to withdraw early in case  $R \in [1, 1/\lambda]$ . The “bad equilibrium” has been eliminated by the existence of the IMF. Interestingly, the IMF does not need to lend anything, as long as agents know it will be there to prevent a liquidity crises.

The model makes a few strong assumptions.

- The IMF has deep pockets. If the IMF cannot cover all early withdrawals, the bad equilibrium might still exist in the multiple-equilibrium range.
- The IMF and private agents have perfect information about fundamentals.

Advertisement: for a model where the IMF is a large player, has only a limited amount of resources and agents have incomplete information about fundamentals, see Corsetti, Guimaraes and Roubini (2006).

## 5.2 Coordination and the price of debt

Now let’s study a coordination problem that arises when investors are deciding whether they roll over debt or not. The model follows section 2.3.1 of Morris and Shin (2003) which is a simplified version of Morris and Shin (2004).

### 5.2.1 Morris and Shin (2004)

A measure-one continuum of investors chooses whether or not to roll over debt. An investor that does not roll over receives  $\kappa \in (0, 1)$  at  $t = 1$ ,  $\kappa$  can be interpreted as collateral of a firm’s debt. Rolling over yields 1 if the firm (or country) is successful able to repay its debt and 0 otherwise. The firm is successful and able to repay if  $l \leq \theta/z$ ,

where  $l$  is the fraction of individuals that withdraw early,  $\theta$  is the level of ‘fundamentals’ and  $z$  is a positive constant.

At  $t = 1$ , agents receive a signal  $x_i = \theta + \varepsilon_i$ , where  $\varepsilon_i \sim N(0, \sigma)$  and for agents  $i$  and  $j$ ,  $E(\varepsilon_i \varepsilon_j) = 0$ . We will think of  $\sigma \rightarrow 0$  so that we don’t need to worry about their prior information.

Equilibrium is characterized by 2 thresholds:

- An agent chooses to roll over if  $x_i > x^*$ .
- The firm is successful if  $\theta \geq \theta^*$ .

An agent with signal  $x_i = x^*$  considers that

$$\begin{aligned} \Pr(\theta > \theta^*) &= \Pr(x^* - \varepsilon_i > \theta^*) \\ &= 1 - \Phi\left(\frac{\theta^* - x^*}{\sigma}\right) \\ &= \Phi\left(\frac{x^* - \theta^*}{\sigma}\right) \end{aligned}$$

The agent is indifferent between withdrawing and rolling it over if

$$\kappa = \Phi\left(\frac{x^* - \theta^*}{\sigma}\right)$$

Moreover consider the case  $\theta = \theta^*$ . Then

$$l = \Pr(x_i < x^*) = \Phi\left(\frac{x^* - \theta^*}{\sigma}\right)$$

At the equilibrium threshold  $\theta^*$ , we know that

$$l = \frac{\theta^*}{z}$$

Hence

$$\theta^* = z\kappa$$

Now let’s say that  $G(\cdot)$  is the prior cdf on  $\theta$ ,  $g(\cdot)$  is the pdf. Then the ex-ante value of debt as a function of  $\kappa$  is given by

$$\begin{aligned} V(\kappa) &= \kappa G(\theta^*) + 1(1 - G(\theta^*)) \\ &= 1 - (1 - \kappa)G(z\kappa) \end{aligned}$$

which implies

$$\frac{\partial V}{\partial \kappa} = G(z\kappa) - z(1 - \kappa)g(z\kappa)$$

Hence an increase in  $\kappa$  affects price of debt in two ways:

1. It increases the value of debt if  $\theta$  happens to be low and the agent chooses not to roll over. That is the term  $G(z\kappa)$ .
2. However, it also increases incentives for early withdrawal and hence increases the threshold  $\theta^* = z\kappa$ , making it more difficult for agents to coordinate on rolling over the debt.

For low values of  $\kappa$ , the overall effect of  $\kappa$  on the ex-ante value of debt is actually ambiguous owing to the coordination channel.

## 6 Expectations and option prices

Currency crises are rare events, so data explicitly relating to them are relatively scarce. But that problem may be overcome using financial price data, which are abundant and reflect expectations about currency devaluations.

### 6.1 Expectations implicit in financial prices

#### 6.1.1 Rose and Svensson (1994)

Rose and Svensson (EER 1994) estimate expectations of changes in the exchange rate before the ERM crisis in 1992. Key questions are: (i) was the crisis expected? (ii) how much of the change on expectations are explained by macroeconomic variables? Their answers are: (i) basically no and (ii) basically nothing.

**Measuring credibility** Notation:  $\delta_t$  is the interest rate differential between a given country and Germany and  $s_t$  is the exchange rate (price of a Deutschmark in domestic currency). Uncovered interest parity implies:

$$\delta_t = \frac{E[\Delta s_t]}{\Delta t}$$

Now, the exchange rate was allowed to fluctuate inside a band. Thus,  $s$  may be written as  $s = x + c$ , where  $c$  is the (log of) the central parity and  $x$  denotes deviations of the spot rate from  $c$ . Expected depreciation can be separated in 2 parts:

$$\frac{E[\Delta s_t]}{\Delta t} \equiv \frac{E[\Delta x_t]}{\Delta t} + \frac{E[\Delta c_t]}{\Delta t}$$

They use 2 measures of realignment expectations:

1. measure is  $\delta_t$ . It assumes  $E[\Delta x_t]/\Delta t = 0$ .
2. Estimates  $E[\Delta x_t]/\Delta t$  with a linear regression:

$$\frac{\Delta x_t}{\Delta t} = \sum \alpha_i + \beta \cdot x_t + \gamma \cdot \delta_t + u_t$$

How to interpret it?  $g = 10\%$  may mean an expected realignment of 10% with hazard rate 1/year or an expected realignment of 1% with hazard rate 10/year...

**Fundamentals?** Rose and Svensson try to assess the impacts of macroeconomic fundamentals on some macroeconomic variables with:



- a regression of  $g$  in a set of macroeconomic variables (with country and time dummies).
- the estimation of a VAR.

They find that macroeconomic variables have very little impact on credibility.

**What to take out of this?** Positive results would be hard to be explained given the lack of a structural model and, especially, the endogeneity of many of the regressors. The authors claim that it is more difficult to dismiss negative results — if there was some important relation between those macroeconomic variables and the credibility of the exchange rate parity, their analysis should have detected it.

The analysis is very simple. Both the estimation of credibility and the assessment of the importance of fundamentals are quite atheoretical. How can it be improved?

## 6.2 Option prices

The exchange rate risk in a pegged regime depends on the probability that the peg will be abandoned and on the expected size of a consequent currency devaluation. To a first order approximation, the forward premium (or the interest rate differential) is roughly the product of these two variables. However, observing the forward premium alone does not permit individual identification of the probability of a devaluation and its expected magnitude: a forward premium of 3% a year may refer to an expected devaluation of 30% with probability 10% a year, or an expected devaluation of 5% with probability 60% a year, and so on.

Options are a richer source of data because they provide information about the probability density of the exchange rate at different points. So it is possible to disentangle the “thickness of the tail of the distribution” (probability of a devaluation) and the “distance from the tail to the center” (the expected magnitude of a devaluation).

To give a simple intuition for identification, suppose the price of an asset tomorrow will be 1 with probability  $1 - p$  and 3 with probability  $p$ . In a risk-neutral world, a call option with strike price 1 costs  $2p$ , a call option with strike price 2 costs  $p$ . If the probability of a devaluation ( $p$ ) increases, both options get more expensive but the ratio of their prices remains equal to 2. If the magnitude of the devaluation increases from 3 to 4, the option with strike price 1 will cost  $3p$ , a call option with strike price 2 will cost  $2p$  — the ratio changes.

### 6.2.1 The Black and Scholes model

Before we start studying the options, let's take a look at the basic asset pricing model.

The Black and Scholes model assumes that the asset follows:

$$\frac{dS}{S} = \mu \cdot dt + \sigma \cdot dZ$$

where:

- $\mu$  is the instantaneous expected return on the asset (ex-dividend);
- $\sigma$  is the instantaneous variance conditional on no jumps and
- $Z$  is a standard Wiener process;

The B&S model is the benchmark model in financial applications. According to the model, the distribution of returns on an asset is log-normal. The data, however, does not fully comply with the B&S formula. In particular, the tails of the distribution are too thick.

The B&S price of an (European) option depends on observed variables (interest rate, spot value of the asset, strike price, time to maturity) and the volatility. With the price of an option, we can calculate its implicit volatility. Usually, we observe that the implicit volatility increases with  $|S - X|$ . This generates the so called *volatility smiles*.

### 6.2.2 Campa, Chang and Refalo (2002)

Campa, Chang and Refalo (JDE 2002) use options to measure the credibility of Brazilian exchange rate regime. Among financial prices, options are better sources of information on the expectations about a peg than future prices (or the interest rate differential) because their value at maturity is nonzero only if the exchange rate goes beyond a certain level (the strike price). So, if there is data on options of different strike classes, there is information about the probability density of the exchange rate at different points, and it is possible to uncover more information about the expectations on the path of the exchange rate. For example, it is possible to identify the probability that the currency peg will be abandoned and the expected magnitude of a devaluation (conditional on its occurrence).

Campa *et al* employ an interesting non-parametric approach that builds on the seminal contribution of Breeden and Litzenberger (1978).

**Market expectations and option prices** Under risk neutrality, the price of a call with strike price  $K$  and expiration date  $T$  is:

$$C_{K,T} = \frac{1}{1+i_t} \int_K^\infty (S_T - K) \cdot f(S_T) \cdot dS_T$$

Differentiating if we respect to  $K$ , we get:

$$\frac{\partial C_{K,T}}{\partial K} = -\frac{1}{1+i_t} \cdot \left[ \int_K^\infty f(S_T) \cdot dS_T \right] = -\frac{1}{1+i_t} \cdot [1 - F(K)]$$

Differentiating again, we get:

$$\frac{\partial^2 C_{K,T}}{\partial K^2} = \frac{1}{1+i_t} \cdot f(K)$$

Intuition:

A one-dollar increase in the strike price decreases the value of the call by (the present value of) an amount equal to the probability that the option will finish “in-the-money”. The higher the probability the option finishes in the money, the more likely a one-dollar increase in the strike price will matter to the option holder and the greater the decrease in the option price.

Thus, the second partial derivative of the option price with respect to  $K$  yields the probability density function of the exchange rate at date  $T$ .

The probability functions derived from option prices are the so called “risk-neutral” probabilities. They can differ from the real pdf’s due to risk considerations, but nevertheless they reveal important information about expectations about an asset.

**Estimation** If we had a continuous of options (or, if we had a lot), we could just evaluate numerically the derivatives. The available data is definitely not enough for that.

We could do some interpolation (e.g., spline) and calculate the pdf. Problem: the call prices are not always a convex function (even without interpolation). We do not want negative pdf’s.

Methodology Campa et al employ: they obtain the implied volatility of each option as a function of the strike price. That yields a “volatility smile”. Then, they transform it into a continuous call price function that is twice-differential in strike — which can be done either by fitting the implied volatility as a quadratic function or by some cubic spline interpolation. Having the price of a call option as a continuum function of the strike prices, we apply the formula and get the densities.

**Again, fundamentals?** They ask the question: can realignment “intensity” be explained by the usual macro variables? They regress their measure of intensity of devaluation in a

bunch of macro variables. The intensity measure is the following:

$$G(T) = \int_{\bar{S}}^{\infty} (S_T - \bar{S}) \cdot f(S_T) \cdot dS_T$$

which implies:

$$G(T) = C_{\bar{S}, T} \cdot (1 + i_T)$$

Results: the level of international reserves is the only significant variable in the regression (and it is endogenous). They conclude that results are consistent with past evidence: “macroeconomic variables are largely unable to explain intertemporal movements in realignment risk”.

There is no theory behind their regression. Should the macro variables impact probability, expected magnitude, or both?

### 6.2.3 Bates (1991)

The question David Bates is asking is: was the crash of '87 expected? That is: were put options too expensive prior to the crash?

In the first section of the paper, Bates examines the skewness of the implicit distribution. He finds that in the year leading up to the crash the probability of a fall was higher than the probability of a large increase. I will jump to the second section in which he presents a model (actually very similar to Merton, 1976) and estimates its parameters implicit in the prices of options.

**The model** The asset (in this specific case, the S&P 500 index) is assumed to follow:

$$\frac{dS}{S} = (\mu - \lambda \cdot k) \cdot dt + \sigma \cdot dZ + k \cdot dq$$

where:

- $\mu$  is the instantaneous expected return on the asset (ex-dividend);
- $\sigma$  is the instantaneous variance conditional on no jumps;
- $Z$  is a standard Wiener process;
- $k$  is the random percentage jump conditional on its occurrence. It is lognormally distributed:

$$- \ln(1 + k) \sim N(\gamma - \delta^2/2, \delta^2) \equiv N(\gamma', \delta^2).$$

$$- E(k) \equiv \bar{k} = e^\gamma - 1$$

- $\lambda$  is the hazard rate of the Poisson event and
- $dq$  is the Poisson counter:  $\Pr(dq = 1) = \lambda \cdot dt$

His proposition 2 shows that contingent claims are priced as if investors were risk-neutral and the asset price followed a similar jump diffusion (page 1025) with different parameters. Saying differently, by estimating the above model, we are obtaining the risk-adjusted parameters.

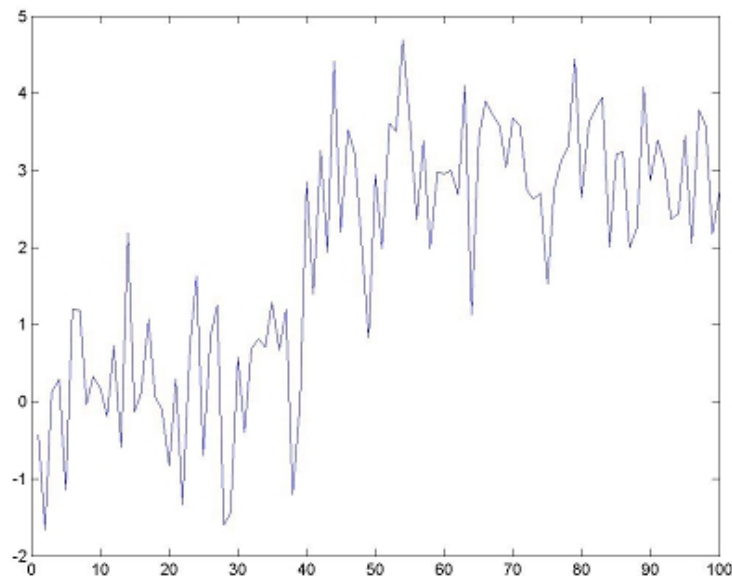


Figure 10: Example of a diffusion path

The asset follows a Black and Scholes diffusion path almost all the time. Sometimes, a Poisson event happens (hazard rate is  $\lambda$ ) and there is a discrete jump.

The parameters of this model are:

- the volatility  $\sigma$ ;
- the hazard rate  $\lambda$ ;
- the mean of the jump  $\gamma$ ; and
- the standard deviation of the jump  $\delta$ .

How changes in each parameter changes the distribution of probability of an European option:

- $\sigma$ : increases standard deviation of the no-jump scenario;
- $\lambda$ : increases the weight of the jump scenario;
- $\gamma$ : pushes the part of the pdf that refers to the jump scenario further away from the no-jump scenario;
- $\delta$ : increases standard deviation of the jump scenario.

How each parameter influences the prices of call options?

**Estimation** The model yields a closed form solution for the price of a call option (equation 11 in the paper). Then, it is assumed that the observed price of an option equals the corresponding model price plus an additive disturbance term (it could be a multiplicative error term also, both have some inconveniences).

Then, a cross-sectional data sample with identical maturities was used and implicit parameters were estimated via non-linear least squares for all days in the sample. The parameters are not constrained to be constant over time.

**Results** It is hard to argue that the estimation succeeded in making a clear distinction between the probability and the expected magnitude of the devaluation (figure 6). It seems that the estimation is picking up something happening at the tail, but it really can't tell what is probability and what is magnitude.

Figure 7 shows the "price of risk" ( $\lambda.k$ ). We can see that  $\lambda.k < 0$  especially from June-1987 to August-1987 (dates are not shown in the horizontal axis, June to August is the period with higher risk of a jump down).

Interestingly, in the 2 months right before the crash, the risk of a jump implicit in option prices is much smaller. Even in the Friday right before the crash, there is no sign of risks of an immediate collapse of stock prices.

#### 6.2.4 Guimaraes (2007)

Time for me to talk about my own work!

Guimaraes (2007) presents procedure for testing whether currency crises depend on sunspots or are triggered when the overvaluation hits a threshold.

The idea is the following: if crises are triggered by currency overvaluation crossing a threshold, the expected magnitude of a devaluation, conditional on its occurrence, is equal to the threshold value, which may differ substantially from the unconditional expected

currency overvaluation. On the other hand, if crises are triggered by sunspots, uncorrelated with the economic variables that determine the exchange rate in a floating regime, then the expected magnitude of a devaluation conditional on its occurrence is similar to the unconditional expected currency overvaluation.

The probability and the expected magnitude of a devaluation are not observable but can be estimated using data on exchange rate options. Guimaraes(2007) identifies the probability and expected magnitude of a devaluation of Brazilian *Real* in the period leading up to the end of the Brazilian pegged exchange rate regime and contrasts the estimates to the predictions from a simple model of currency crises under different assumptions about the trigger.

**The model** As in Bates (1991), a parametric approach is employed. The asset pricing model is the following:

Denote by  $S$  the exchange rate and  $s$  its logarithm. Initially, the exchange rate follows a standard Brownian motion with low volatility:

$$ds = \mu_1 dt + \sigma_1 dX$$

The pegged regime may be abandoned at any time. The interruption is a Poisson event with hazard rate  $\lambda$ . It leads to a discrete jump in the exchange rate and to a new diffusion process, assumed to last forever.

The jump is constant ( $k$ ):

$$\frac{S^{after}}{S^{before}} = (1 + k)$$

The floating regime is described by a Brownian motion with drift and higher volatility:

$$ds = \mu_2 dt + \sigma_2 dX$$

**Results** The empirical results unveil completely different patterns for the probability and expected magnitude of a devaluation (conditional on its occurrence). The probability was volatile and mostly driven by contagion from external crises, as the Asian and Russian crises triggered by far the greatest increases in the probability that the peg would be abandoned. In contrast, the expected magnitude was stable and entirely unaffected by the Russian episode.

In addition, these data suggest that the Asian and Russian crises negatively impacted the Brazilian shadow exchange rate. They explicitly show that the crises coincided with both the greatest increases in the risk of a devaluation in Brazil and the largest depreciations of other Latin American currencies, like the Mexican Peso. Since the crises were

fairly exogenous to the Latin American economies, it is natural to assume that if the Brazilian currency were allowed to float, it would also have depreciated.

**Conclusion** The empirical findings favour thresholds and learning over sunspots.

### 6.2.5 Parametric $\times$ non-parametric method

Some advantages of the parametric method:

- it imposes a structure that makes sense economically.
- interpretation of the results is immediate.
- in the non-parametric case, you need to impose some structure anyway (quadratic, interpolation).
- it is good if there is not much data (market is not so liquid), and if the model is appropriate.

Some disadvantages of parametric method:

- it imposes a structure that may differ from the true data generating process.
- the parametric model is not exactly what you end up estimating (as you vary  $\lambda$ 's and  $\mu$ 's).
- the non-parametric approach may yield accurate results if we have very good data (or if we do not have a good model).

### 6.2.6 Extensions

Jondeau and Rockinger (2000) describe alternative methods to infer risk-neutral densities. Their paper brings a collection of techniques. It is worth discussing a few examples.

**Stochastic volatility** The diffusion process is the following:

$$\begin{aligned}\frac{dS_t}{S_t} &= \mu \cdot dt + \sqrt{\nu_t} \cdot \sigma \cdot dW_1 \\ d\nu_t &= \kappa(\theta - \nu_t)dt + \gamma \cdot \sqrt{\nu_t} \cdot dW_2\end{aligned}$$

Parameters:

- $\theta$ : long-run volatility;



- $\kappa$ : mean-reversion speed;
- $\gamma$ : volatility of the volatility.

The instantaneous volatility  $\nu_t$  is not exactly a parameter, is the realization of a random variable, but it is natural to estimate it as well.

**Sum of log-normals** Another way to imposing some structure in the probability density functions is to consider that the option price is a mixture of M log-normal densities. That is:

$$C_t = e^{-rT} \sum_{i=1}^M \alpha_i \int_{S_t=K}^{\infty} (S_\tau - K) \cdot l(S_\tau; \mu_i, \sigma_i) \cdot dS_\tau$$

That gives us a formula not-much more complicated than the Black-Scholes formula and we can estimate the implicit parameters.

Now, given that we want to impose some structure, what is good about this one? Or, which kind of model could yield some distribution similar to this?

**Stochastic interest rate** Bakshi, Cao and Chen (1997) present a model with stochastic volatility, jumps (hazard rate is given by Poisson distribution and the size of the jump is logn-normal) and *stochastic interest rates*. They estimate the implicit parameters of the model and check what matters.

## 7 Identification through heteroskedasticity

Rigobon and Sack (2004) use identification through heteroskedasticity to estimate the impacts of monetary policy on asset prices.<sup>5</sup> We have to deal with two main problems:

1. endogeneity: the policy decision may be influenced by simultaneous changes in asset prices.
2. omitted variables: a number of other variables may have an impact on the policy decision and on asset prices.

The following system of equations captures these features.

$$\Delta s_t = \alpha \Delta i_t + z_t + \eta_t \quad (36)$$

$$\Delta i_t = \beta \Delta s_t + \gamma z_t + \varepsilon_t \quad (37)$$

Where  $\Delta i_t$  is the change in the 1-year interest rate,  $\Delta s_t$  is the change in the spot exchange rate,  $z_t$  is an omitted variable,  $\varepsilon_t$  is a monetary policy shock and  $\eta_t$  is a shock to the asset price.

If we run OLS on equation 36, we get the following expression for the expected  $\hat{\alpha}$ :

$$E(\hat{\alpha}) = \alpha + (1 - \alpha\beta) \frac{\beta\sigma_\eta + (\beta + \gamma)\sigma_z}{\sigma_\varepsilon + \beta^2\sigma_\eta + (\beta + \gamma)^2\sigma_z}$$

which means that the estimator for  $\alpha$  is biased unless  $\sigma_\varepsilon/\sigma_z \rightarrow \infty$  and  $\sigma_\varepsilon/\sigma_\eta \rightarrow \infty$ .

Now, let's assume that  $z_t$ ,  $\varepsilon_t$  and  $\eta_t$  have no serial correlation and are uncorrelated with each other. Let's divide the sample, consisting on values of  $\Delta s$  and  $\Delta i$ , in two subsamples: the subset  $F$  corresponds to the dates where the FOMC meets, and the subset  $N$  corresponds to dates with no meeting. The number of observations in each of the two sets is denoted by  $T_F$  and  $T_N$ , respectively. The key assumption in Rigobon and Sack (2004) is that the variance of the shock to the interest rate ( $\varepsilon_t$ ) in the dates belonging to the set  $F$  is higher than the variance of the shock to the interest rate in the dates belonging to the set  $N$ , while the variances of  $\eta_t$  and  $z_t$  are the same:

$$\begin{aligned} \sigma_\varepsilon^F &> \sigma_\varepsilon^N \\ \sigma_\eta^F &= \sigma_\eta^N \\ \sigma_z^F &= \sigma_z^N \end{aligned}$$

The traditional way of analyzing the impact of monetary policy decisions (the so-called event study approach) is to consider that unexpected changes in the policy rate are exogenous and use those to estimate equation 36 (which basically means assuming  $\sigma_\varepsilon/\sigma_z \rightarrow \infty$

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<sup>5</sup>See also Rigobon (2003).

and  $\sigma_\varepsilon/\sigma_\eta \rightarrow \infty$ ). Rigobon and Sack (2004) show that such strong assumptions are not needed: with the above assumptions on heteroskedasticity, we can consistently estimate  $\alpha$ . The intuition is the following: in dates where the FOMC meets, there is a shock to equation 37,  $\sigma_\varepsilon$  increases, but there are no shocks to other variables. So, the overall relation between  $\Delta s$  and  $\Delta i$  should be different between the two subsamples,  $F$  and  $N$ . The figures at page 1557-1559 illustrate this idea.

Solving for the reduced form of equations 36 and 37, we reach:

$$\Delta i_t = \frac{1}{1 - \alpha\beta} [(\beta + \gamma) z_t + \beta\eta_t + \varepsilon_t] \quad (38)$$

$$\Delta s_t = \frac{1}{1 - \alpha\beta} [(1 + \alpha\gamma) z_t + \eta_t + \alpha\varepsilon_t] \quad (39)$$

Define  $\Delta x_t = [\Delta i_t, \Delta s_t]'$ . The covariance matrices in each subsample are  $\Omega_F = E(\Delta x_t \Delta x_t' | t \in F)$ , and  $\Omega_N = E(\Delta x_t \Delta x_t' | t \in N)$ . Defining  $\Delta\Omega \equiv \Omega_F - \Omega_N$ , we get:

$$\Delta\Omega = \frac{\sigma_\varepsilon^F - \sigma_\varepsilon^N}{(1 - \alpha\beta)^2} \begin{bmatrix} 1 & \alpha \\ \alpha & \alpha^2 \end{bmatrix} \quad (40)$$

In words, because  $\sigma_\varepsilon^F \neq \sigma_\varepsilon^N$ , the covariance matrix of  $\Delta s$  and  $\Delta i$  is not the same in subsets  $F$  and  $N$  and therefore it is possible to identify  $\alpha$  by looking at the difference in the covariance matrix in the two different subsamples.

Now, consider the following variables:

$$\begin{aligned} \Delta I &\equiv \left[ \frac{\Delta i'_F}{\sqrt{T_F}}, \frac{\Delta i'_N}{\sqrt{T_N}} \right]' \\ \Delta S &\equiv \left[ \frac{\Delta s'_F}{\sqrt{T_F}}, \frac{\Delta s'_N}{\sqrt{T_N}} \right]' \\ w_i &\equiv \left[ \frac{\Delta i'_F}{\sqrt{T_F}}, \frac{-\Delta i'_N}{\sqrt{T_N}} \right]' \\ w_s &\equiv \left[ \frac{\Delta s'_F}{\sqrt{T_F}}, \frac{-\Delta s'_N}{\sqrt{T_N}} \right]' \end{aligned}$$

A major result in Rigobon and Sack (2004) is that  $\alpha$  can be consistently estimated by a standard instrumental variables approach with those novel instruments,  $w_i$  and  $w_s$ . Why are  $w_i$  and  $w_s$  valid instruments for  $\Delta I$  at equation 36? First, the instrument  $w_i$  is correlated with  $\Delta I$  because the bigger variance in the Copom dates implies that the positive correlation between  $(\Delta i'_F/\sqrt{T_F})$  and  $(\Delta i'_N/\sqrt{T_N})$  more than outweighs the

negative one between  $(\Delta i'_N/\sqrt{T_N})$  and  $(-\Delta i'_N/\sqrt{T_N})$ . Formally:

$$\begin{aligned} \text{plim} \frac{1}{T} w'_i \Delta I &= \frac{1}{T_F} \Delta i'_F \cdot \Delta i_F - \frac{1}{T_N} \Delta i'_N \cdot \Delta i_N \\ &= \frac{1}{(1-\alpha\beta)^2} \sigma_\varepsilon^F - \frac{1}{(1-\alpha\beta)^2} \sigma_\varepsilon^N > 0 \end{aligned}$$

On the other hand,  $w_i$  is not correlated with neither  $z$  nor  $\eta$  because the positive and negative correlation of each part of the vector exactly cancel each other out:

$$\begin{aligned} \text{plim} \frac{1}{T} w'_i z &= \frac{1}{T_F} \Delta i'_F \cdot z_F - \frac{1}{T_N} \Delta i'_N \cdot z_N \\ &= \frac{\beta + \gamma}{1 - \alpha\beta} \sigma_z^F - \frac{\beta + \gamma}{1 - \alpha\beta} \sigma_z^N = 0 \\ \text{plim} \frac{1}{T} w'_i \eta &= \frac{1}{T_F} \Delta i'_F \cdot \eta_F - \frac{1}{T_N} \Delta i'_N \cdot \eta_N \\ &= \frac{\beta}{1 - \alpha\beta} \sigma_\eta^F - \frac{\beta}{1 - \alpha\beta} \sigma_\eta^N = 0 \end{aligned}$$

The paper also shows how we could implement this methodology through GMM. Then, it shows how we could test the following null hypotheses:

1. The assumed information structure is correct. For this, note that equation 40 yields 3 moment conditions and 2 parameters ( $\alpha$  and  $\frac{\sigma_\varepsilon^F - \sigma_\varepsilon^N}{(1-\alpha\beta)^2}$ ).
2. The event study is unbiased (this is a Hausman test).

Results are presented on pages 1567-1572.

## 7.1 Evidence on default costs

Hebert and Schreger (2017) employ the method of identification through heteroskedasticity to estimate the effects of changes in the probability of default on the value of assets in Argentina.

The source of exogenous variation is the legal battle between creditor holdouts and Argentina in US courts, described in details in Hebert and Schreger (2017). It all started in December 2001 when Argentina defaulted on over \$100 billion in external sovereign debt. The exchange rate was devalued by around 75 percent. In January 2005, Argentina presented an offer to creditors that was accepted by holders of \$62 billion of debt. S&P declared the end of Argentina default in June. Argentina fully repaid the IMF in 2006. In December 2010, Argentina offered another bond exchange. Many creditors agreed to

the exchange. Remaining holdouts were owed around \$11 billion. Argentina had then structured 90% of its debt.

Creditors led by NML started a legal battle based on the pari passu clause (equal treatment). There were numerous rulings between December 2011 to July 2014 in the Court for the Southern District of New York, the Court of Appeals for the Second Circuit and the Supreme Court. These rulings are shocks to the probability that Argentina will default on its debt, but are pretty exogenous to the Argentinian economy. The method of identification through heteroskedasticity fits very well here. With the dates of the legal rulings issued by those 3 courts, we can estimate the effect of exogenous shocks to the default probability on asset prices in Argentina.

They find large effects. A 10 percent increase in the probability of default causes a 6 percent decline in the value of Argentine equities and a 1 percent depreciation of a measure of the exchange rate.

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